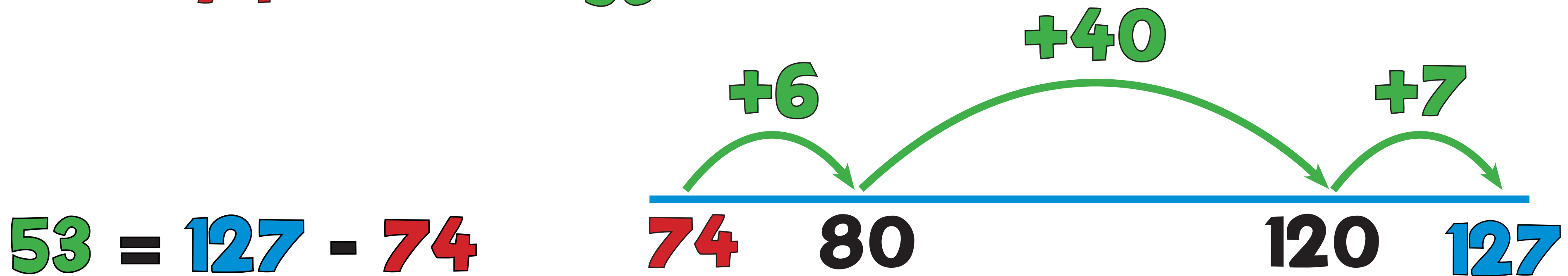
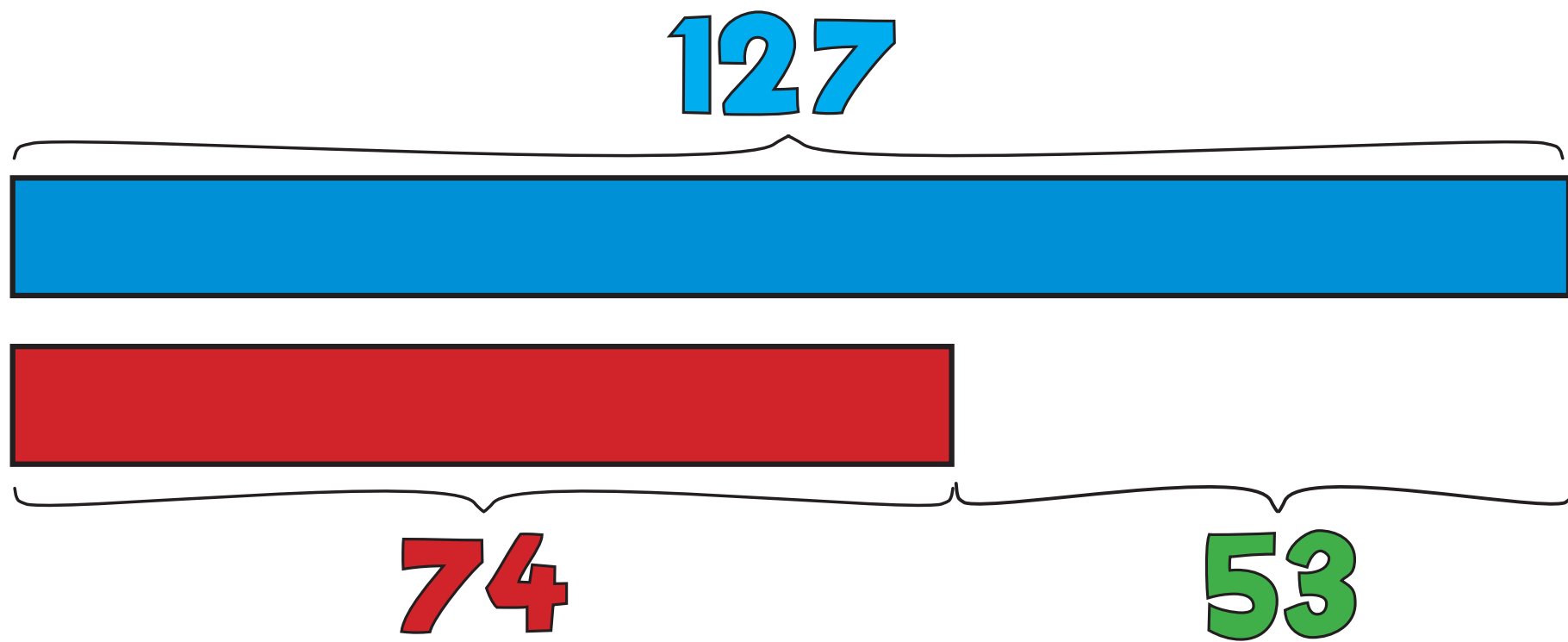
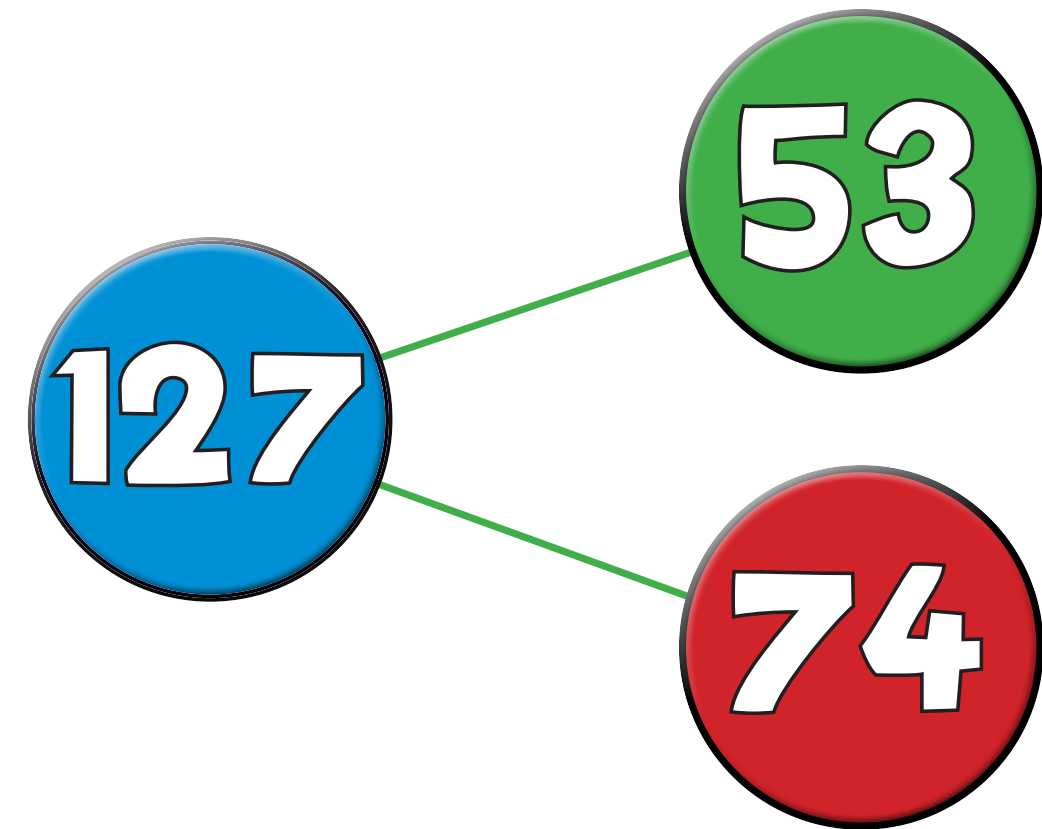
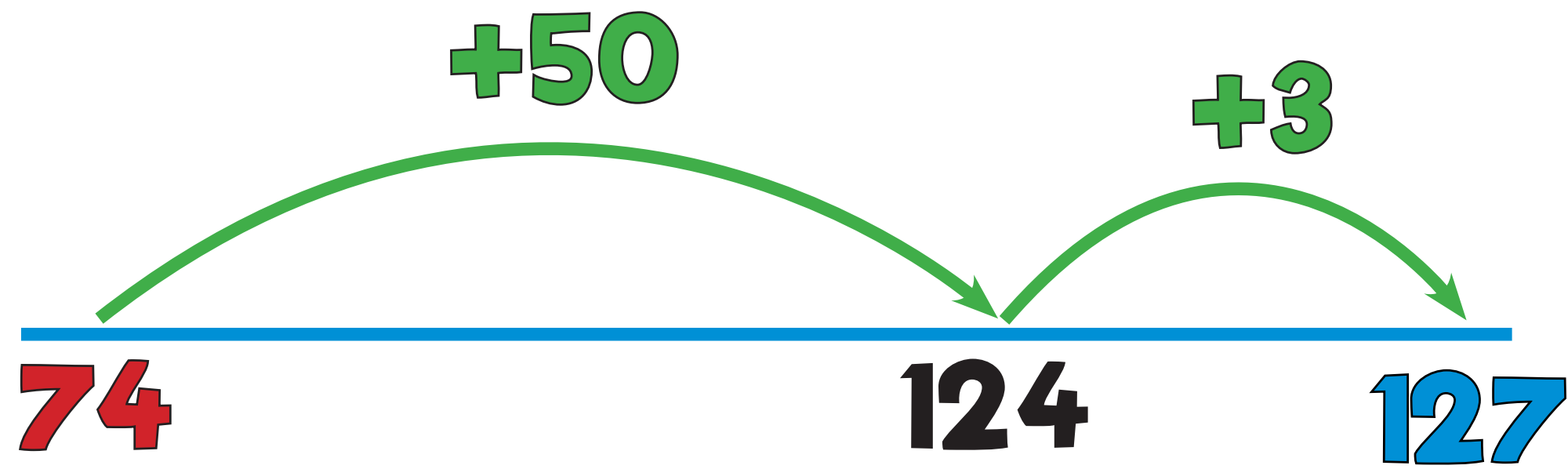
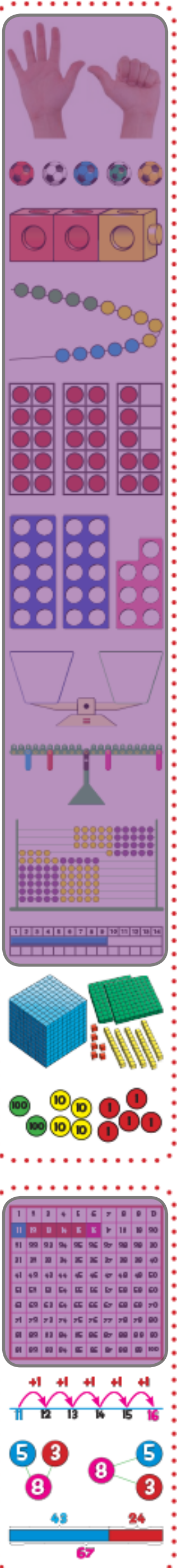


$$127 - 74 = 53$$



$$53 = 127 - 74$$



132 - 56 = 76

1

100	10	1
1	3	2
-	5	6
<hr/>		

Hundreds Tens Ones

2

100	10	1
1	3 ²	2 ¹
-	5	6
<hr/>		

Hundreds Tens Ones

3

100	10	1
1	3 ²	2 ¹
-	5	6
<hr/>		

Hundreds Tens Ones

6

100	10	1
1	3 ¹²	2 ¹
-	5	6
<hr/>		
76		

Tens Ones Tens Ones

4

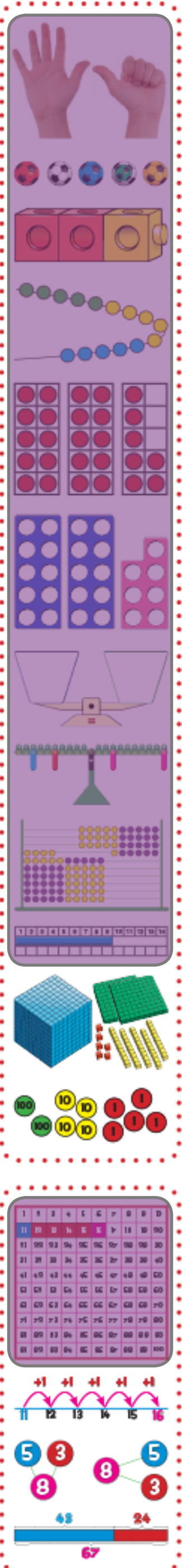
100	10	1
1	3 ²	2 ¹
-	5	6
<hr/>		
6		

Hundreds Tens Ones

5

100	10	1
1	3 ¹²	2 ¹
-	5	6
<hr/>		
6		

Hundreds Tens Ones



$$132 - 56 = 76$$

1

100	10	1
1	3	2
-	5	6
<hr/>		

Hundreds Tens Ones

100 10 10 10 1 1

2

100	10	1
1	3	2
-	5	6
<hr/>		

Hundreds Tens Ones

100 10 10 10 1 1

10 → 1

3

100	10	1
1	3	2
-	5	6
<hr/>		

Hundreds Tens Ones

100 10 10 1 1 1 1 1 1

4

100	10	1
1	3	2
-	5	6
<hr/>		
		6

Hundreds Tens Ones

100 10 10 1 1 1 1 1 1

5

100	10	1
1	3	2
-	5	6
<hr/>		
		6

Hundreds Tens Ones

100 10 10 1 1 1 1 1 1

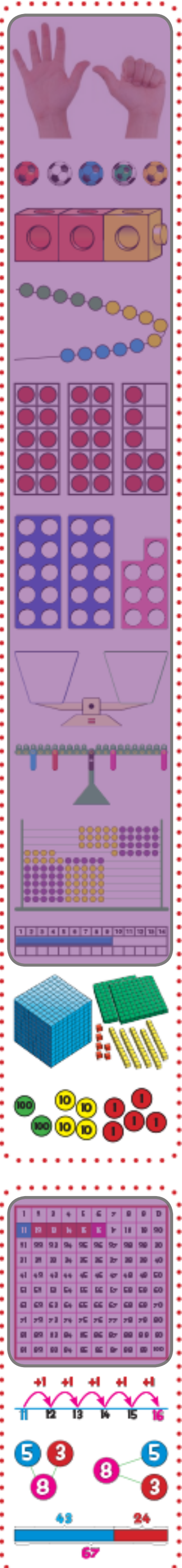
100 → 10 10

6

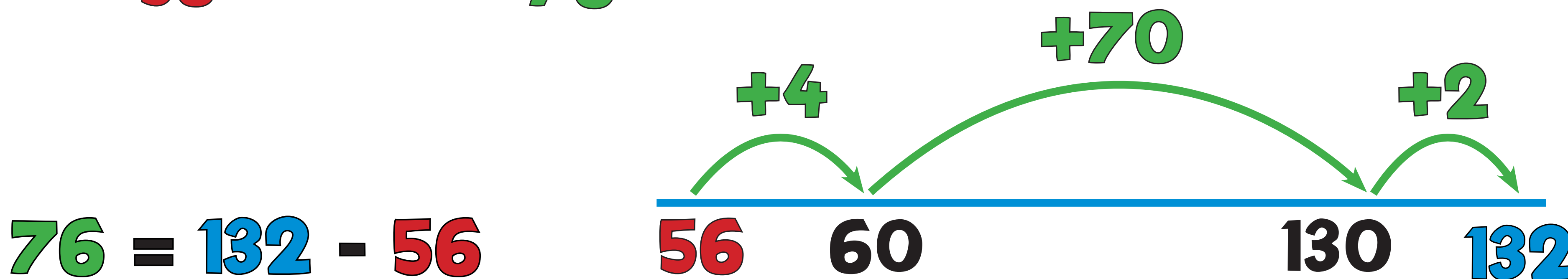
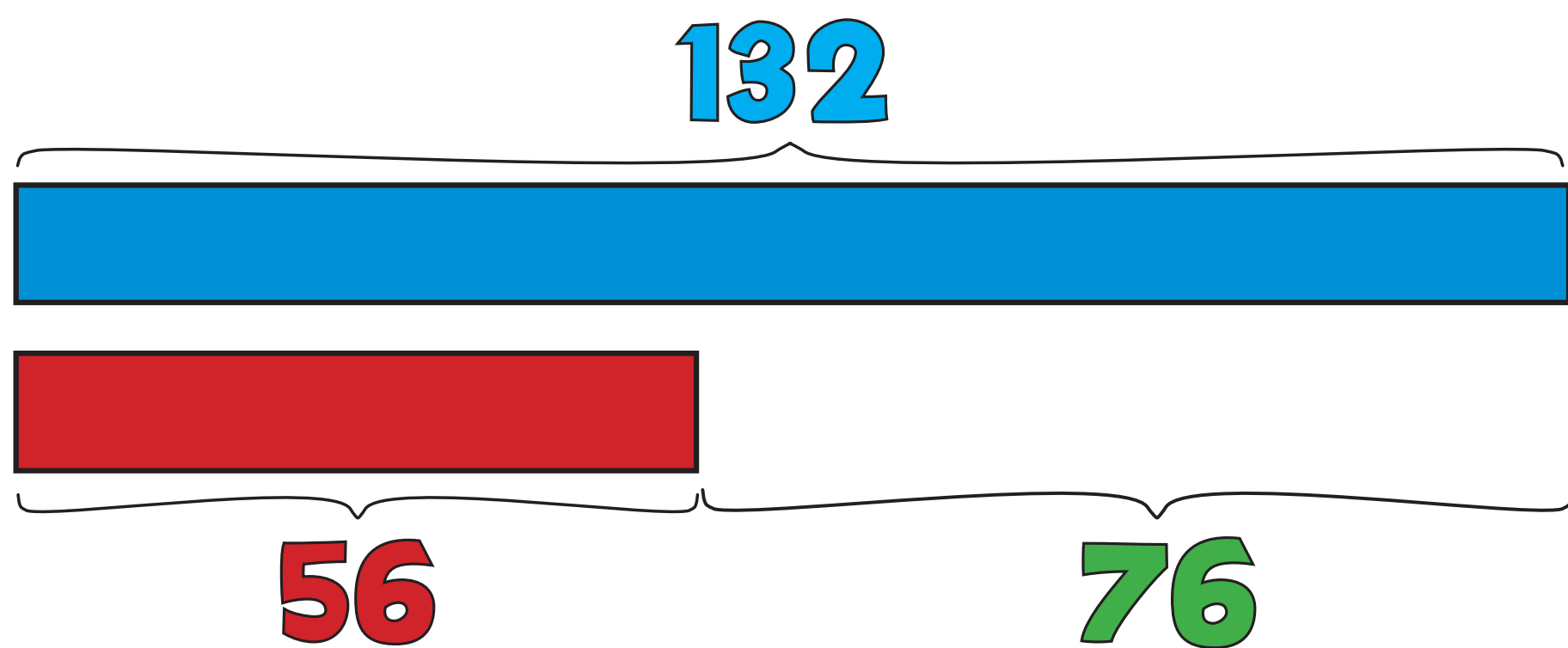
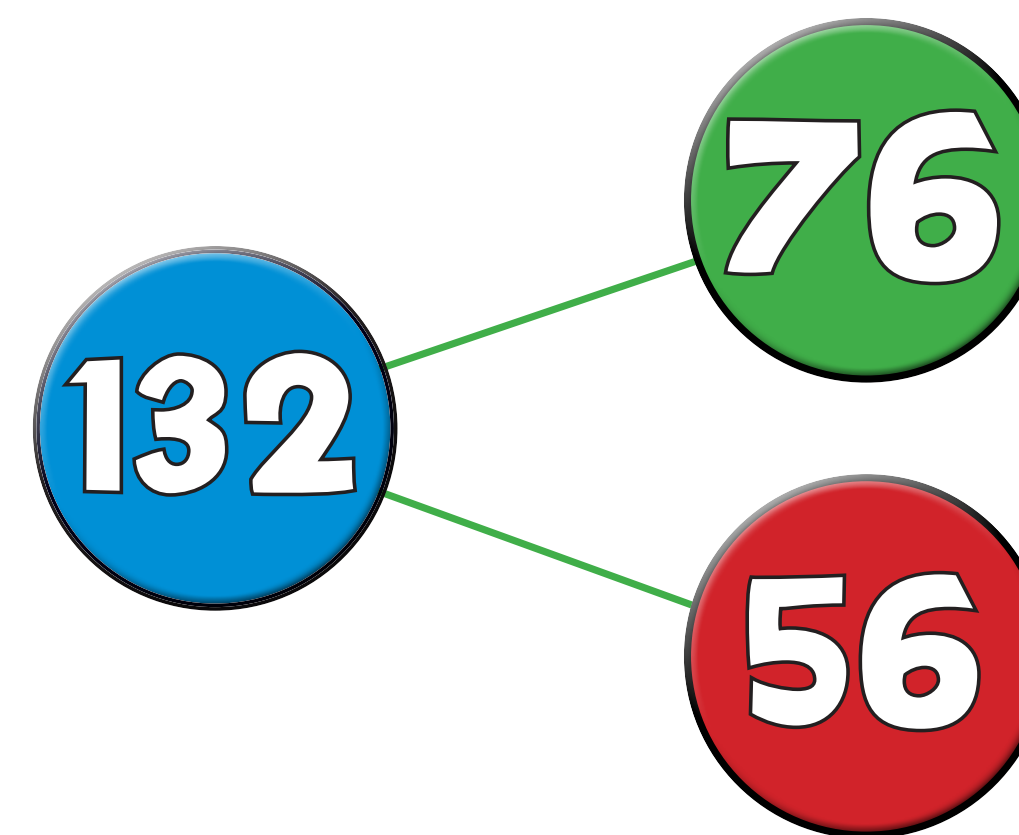
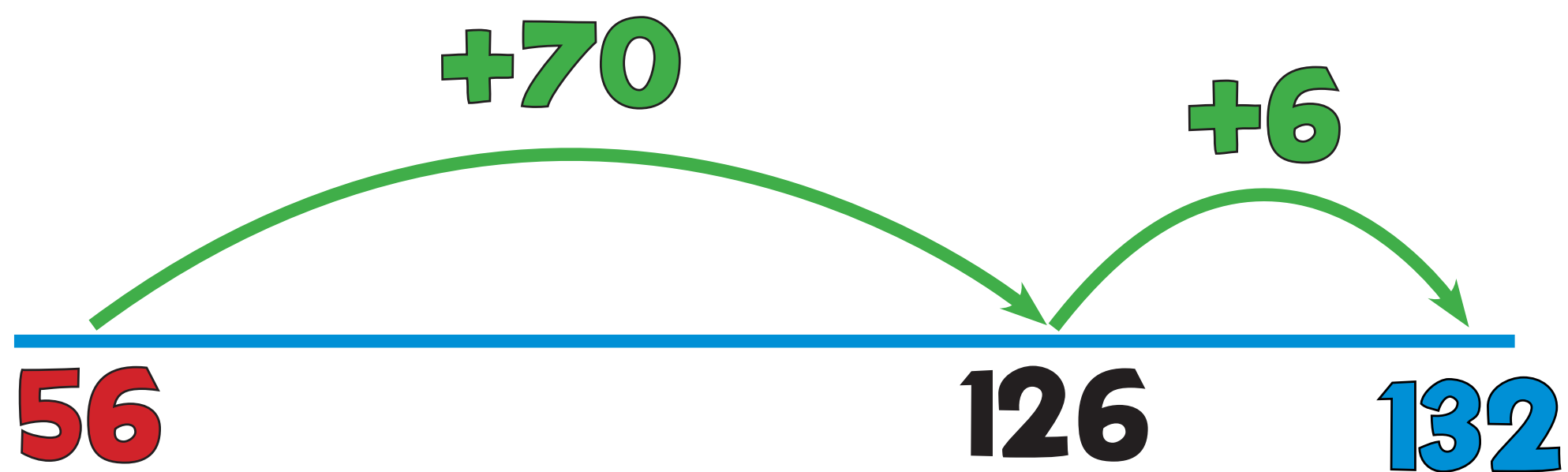
100	10	1
1	3	2
-	5	6
<hr/>		
		6

Tens Ones Tens Ones

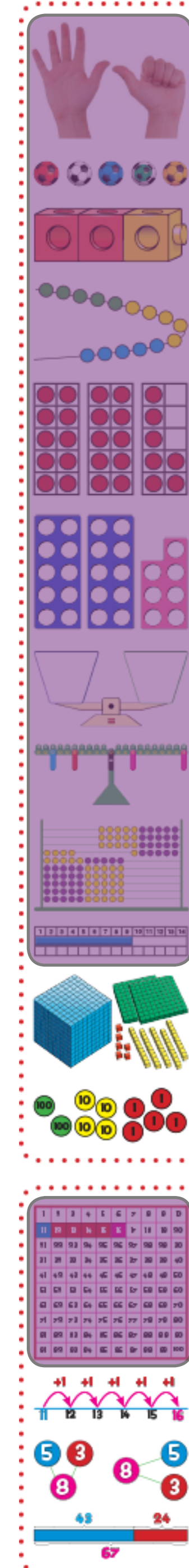
10 10 10 10 10 10 1 1 1 1 1 1



$$132 - 56 = 76$$



$$76 = 132 - 56$$



$$736 - 462 = 274$$

1

100	10	1
7	3	6
-	4	6
<hr/>		

Hundreds Tens Ones

2

100	10	1
6	7	3
-	4	6
<hr/>		

Hundreds Tens Ones

3

100	10	1
6	7	3
-	4	6
<hr/>		
		4

Hundreds Tens Ones

4

100	10	1
6	7	3
-	4	6
<hr/>		
	7	4

Hundreds Tens Ones

5

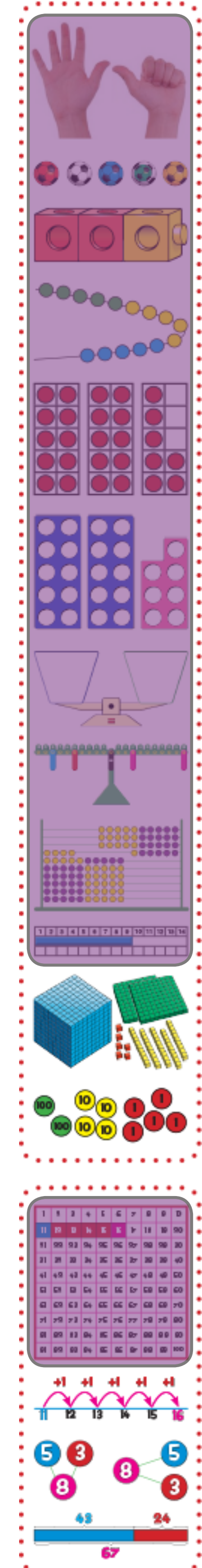
100	10	1
6	7	3
-	4	6
<hr/>		
	7	4

Hundreds Tens Ones

6

100	10	1
6	7	3
-	4	6
<hr/>		
2	7	4

Hundreds Tens Ones



$$723 - 356 = 367$$

1

100	10	1
7	2	3
-	3	5

Hundreds Tens Ones

2

100	10	1
7	2	3
-	3	5

Hundreds Tens Ones

3

100	10	1
7	2	3
-	3	5

Hundreds Tens Ones

4

100	10	1
6	11	13
-	3	5

		7

Hundreds Tens Ones

5

100	10	1
6	11	13
-	3	5

		6

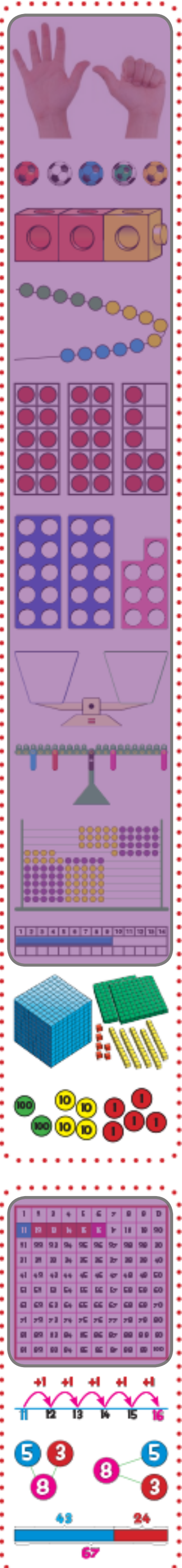
Hundreds Tens Ones

6

100	10	1
6	11	13
-	3	5

		6
		7

Hundreds Tens Ones



CPA
Reasoning

$$723 - 356 = 367$$

Part 2

7

	100	10	1
6	11	1	
7	2	3	
-	3	5	6
<hr/>			
		6	7

Hundreds Tens Ones

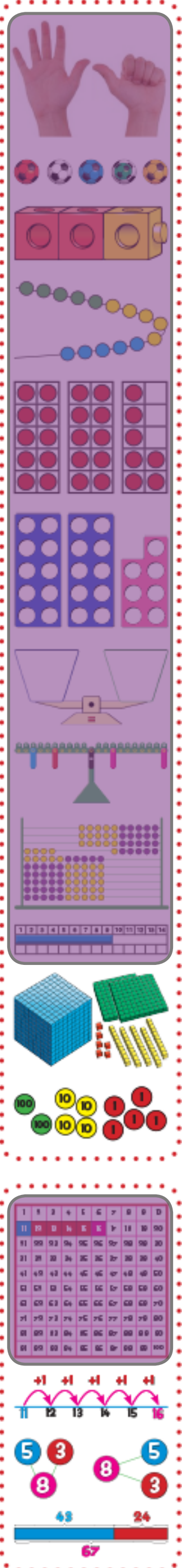
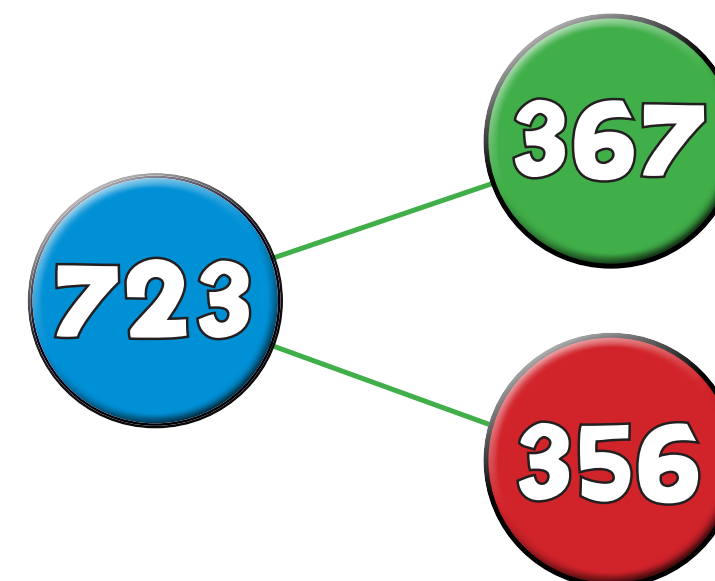
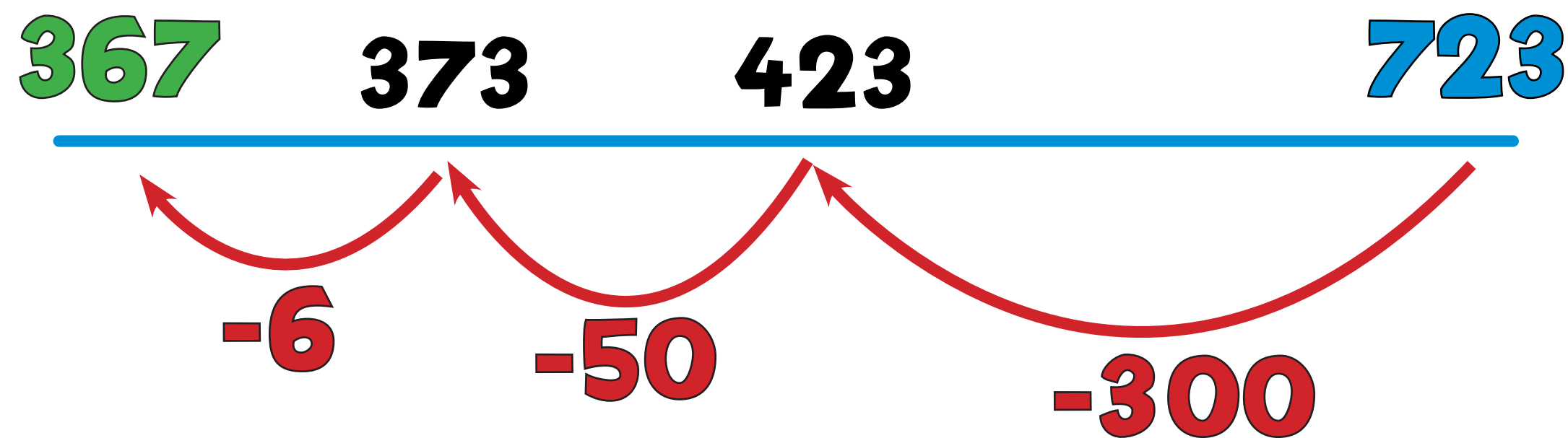
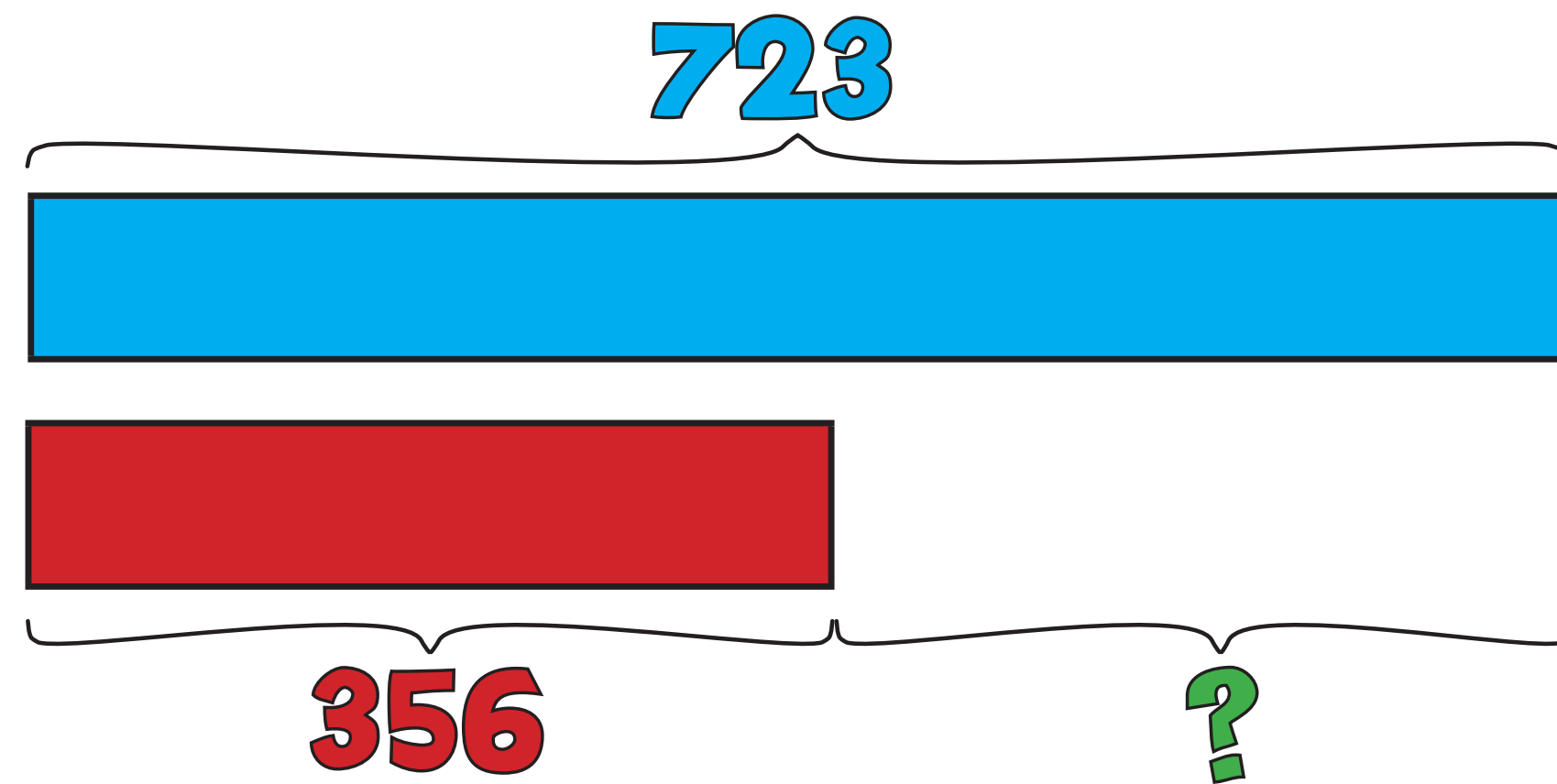
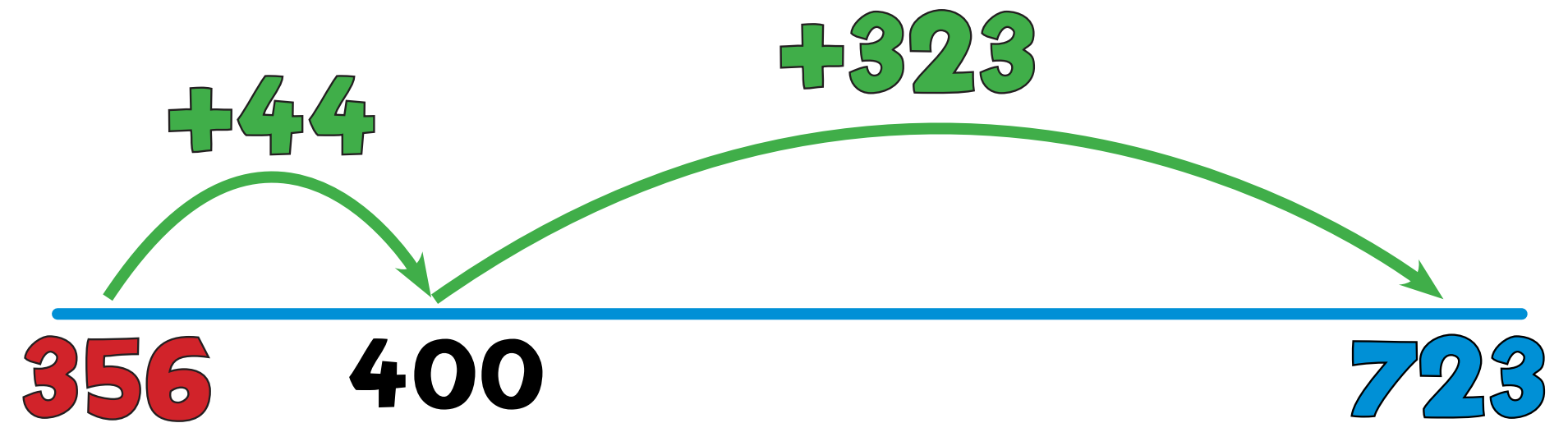
56

8

	100	10	1
6	11	1	
7	2	3	
-	3	5	6
<hr/>			
		3	6
			7

Hundreds Tens Ones

356



1375 - 538 = 837

1

Thousands	Hundreds	Tens	Ones
1000	100	10	1
1	3	7	5
-	5	3	8

2

Thousands	Hundreds	Tens	Ones
1000	100	10	1
1	3	7	5
-	5	3	8

3

Thousands	Hundreds	Tens	Ones
1000	100	10	1
1	3	7	5
-	5	3	8

4

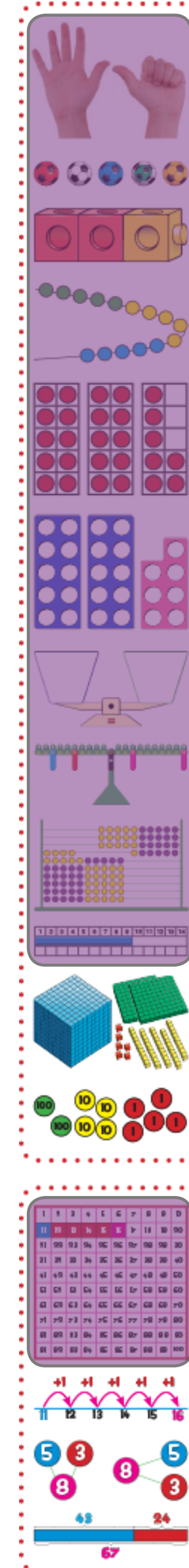
Thousands	Hundreds	Tens	Ones
1000	100	10	1
1	3	7	5
-	5	3	8

5

Thousands	Hundreds	Tens	Ones
1000	100	10	1
1	3	7	5
-	5	3	8

6

Thousands	Hundreds	Tens	Ones
1000	100	10	1
1	3	7	5
-	5	3	8



$$1375 - 538 = 837$$

7

Thousands Hundreds Tens Ones

1000 100 10 1

$$\begin{array}{r} 1375 \\ - 538 \\ \hline 37 \end{array}$$

8

Thousands Hundreds Tens Ones

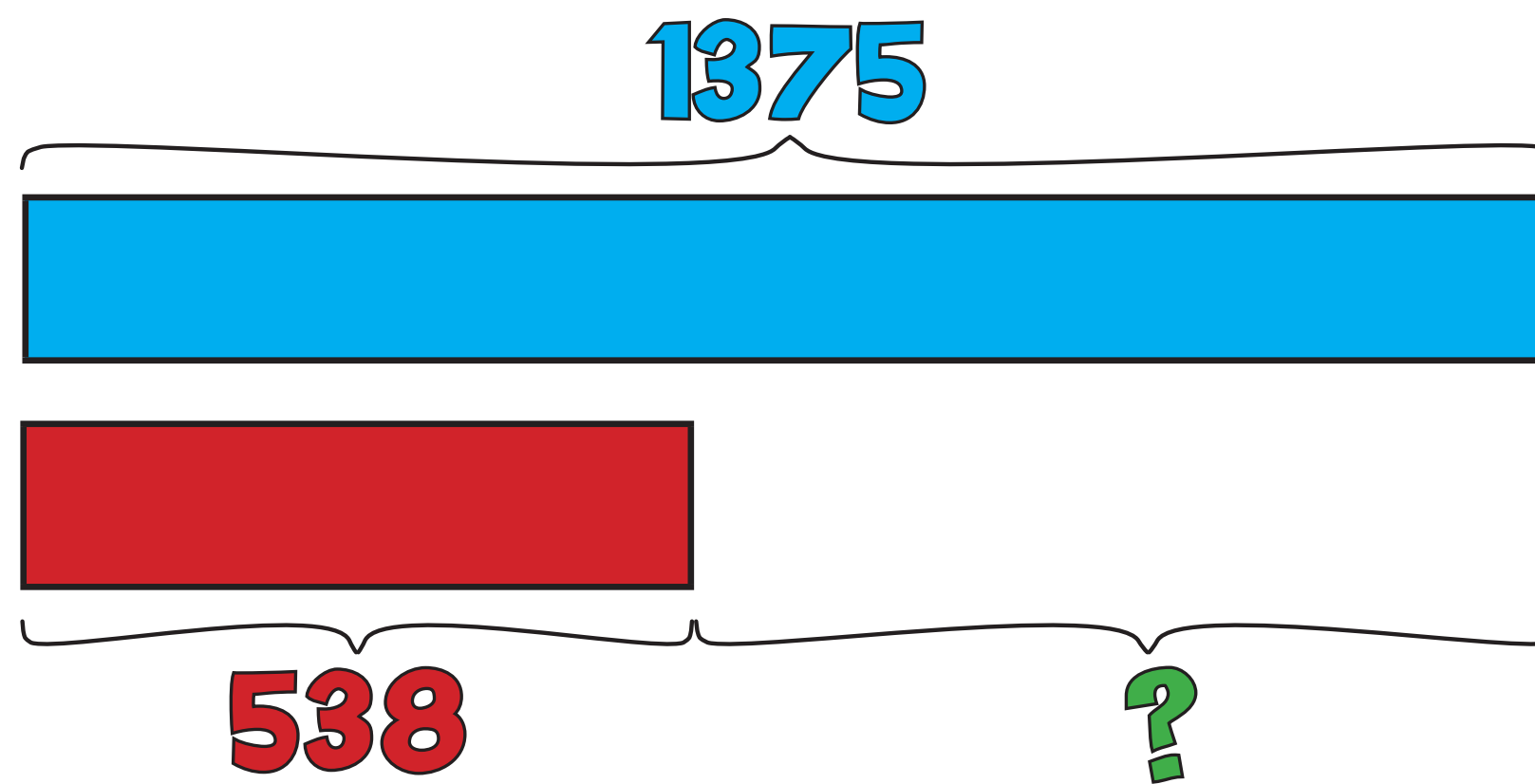
1000 100 10 1

$$\begin{array}{r} 1375 \\ - 538 \\ \hline 37 \end{array}$$

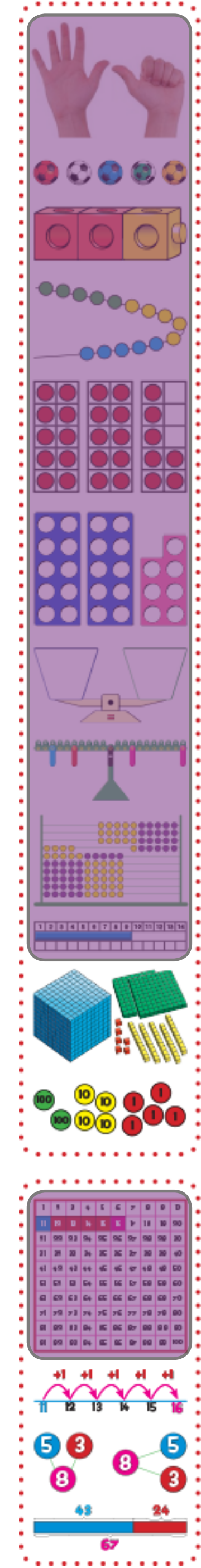
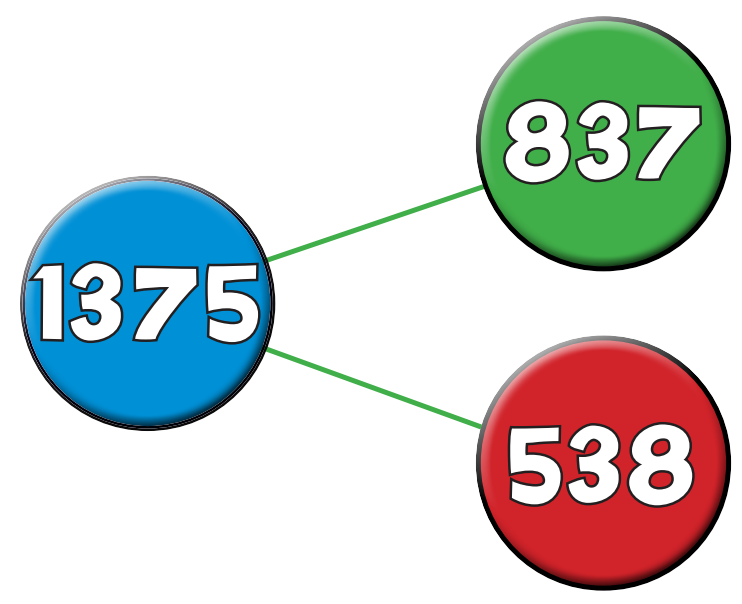
9

Thousands Hundreds Tens Ones

1000 100 10 1

$$\begin{array}{r} 1375 \\ - 538 \\ \hline 837 \end{array}$$


It's a 'take away' image!



$$5042 - 1776 = 3266$$

7

Thousands Hundreds Tens Ones

1000 100 10 1

$$\begin{array}{r} 4 \quad 19 \quad 13 \quad 1 \\ 5042 \\ - 1776 \\ \hline 6 \end{array}$$

6

8

Thousands Hundreds Tens Ones

1000 100 10 1

$$\begin{array}{r} 4 \quad 19 \quad 13 \quad 1 \\ 5042 \\ - 1776 \\ \hline 66 \end{array}$$

76

9

Thousands Hundreds Tens Ones

1000 100 10 1

$$\begin{array}{r} 4 \quad 19 \quad 13 \quad 1 \\ 5042 \\ - 1776 \\ \hline 66 \end{array}$$

76

10

Thousands Hundreds Tens Ones

1000 100 10 1

$$\begin{array}{r} 4 \quad 19 \quad 13 \quad 1 \\ 5042 \\ - 1776 \\ \hline 266 \end{array}$$

776

11

Thousands Hundreds Tens Ones

1000 100 10 1

$$\begin{array}{r} 4 \quad 19 \quad 13 \quad 1 \\ 5042 \\ - 1776 \\ \hline 266 \end{array}$$

776

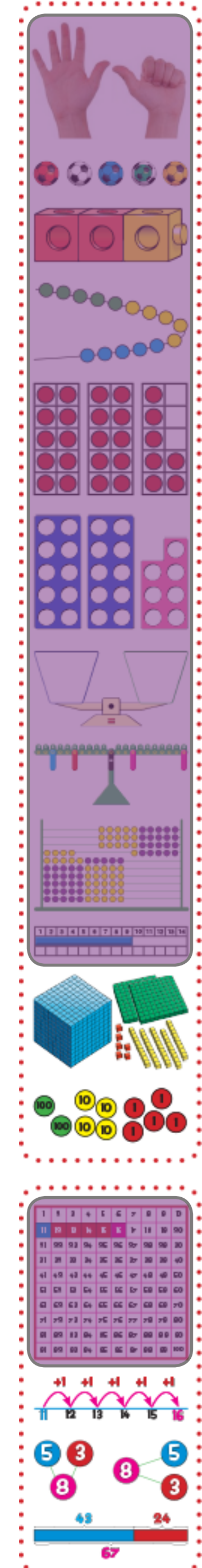
12

Thousands Hundreds Tens Ones

1000 100 10 1

$$\begin{array}{r} 4 \quad 19 \quad 13 \quad 1 \\ 5042 \\ - 1776 \\ \hline 3266 \end{array}$$

1776



$$13.4 - 8.7 = 4.7$$

1

Tens	Ones	Tenths
10	1	1/10
13.4	8	7
<hr/>		
	1	0.1
	1	0.1
	1	0.1

2

Tens	Ones	Tenths
10	1	1/10
13.4	8	7
<hr/>		
	1	0.1
	1	0.1
	1	0.1
	1	0.1
	1	0.1
	1	0.1

3

Tens	Ones	Tenths
10	1	1/10
13.4	8	7
<hr/>		
	1	0.1
	1	0.1
	1	0.1
	1	0.1
	1	0.1
	1	0.1
	1	0.1

4

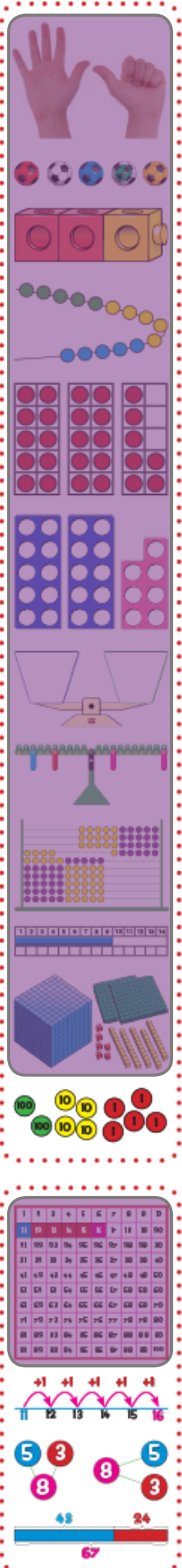
Tens	Ones	Tenths
10	1	1/10
13.4	8	7
<hr/>		
	1	0.1
	1	0.1
	1	0.1
	1	0.1
	1	0.1
	1	0.1

5

Tens	Ones	Tenths
10	1	1/10
13.4	8	7
<hr/>		
	1	0.1
	1	0.1
	1	0.1
	1	0.1
	1	0.1
	1	0.1

6

Ones	Tenths	Ones	Tenths
1	0.1	1	0.1
1	0.1	1	0.1
1	0.1	1	0.1
1	0.1	1	0.1
1	0.1	1	0.1
1	0.1	1	0.1
1	0.1	1	0.1



$$72.43 - 47.85 = 24.58$$

1

10 1 $\frac{1}{10}$ $\frac{1}{100}$

$$\begin{array}{r} 72.43 \\ - 47.85 \\ \hline \end{array}$$

Tens Ones Tenths Hundredths

2

10 1 $\frac{1}{10}$ $\frac{1}{100}$

$$\begin{array}{r} 72.43 \\ - 47.85 \\ \hline \end{array}$$

Tens Ones Tenth Hundredths

3

10 1 $\frac{1}{10}$ $\frac{1}{100}$

$$\begin{array}{r} 72.43 \\ - 47.85 \\ \hline \end{array}$$

Tens Ones Tenth Hundredths

4

10 1 $\frac{1}{10}$ $\frac{1}{100}$

$$\begin{array}{r} 72.43 \\ - 47.85 \\ \hline 8 \end{array}$$

Tens Ones Tenth Hundredths

5

10 1 $\frac{1}{10}$ $\frac{1}{100}$

$$\begin{array}{r} 72.43 \\ - 47.85 \\ \hline 8 \end{array}$$

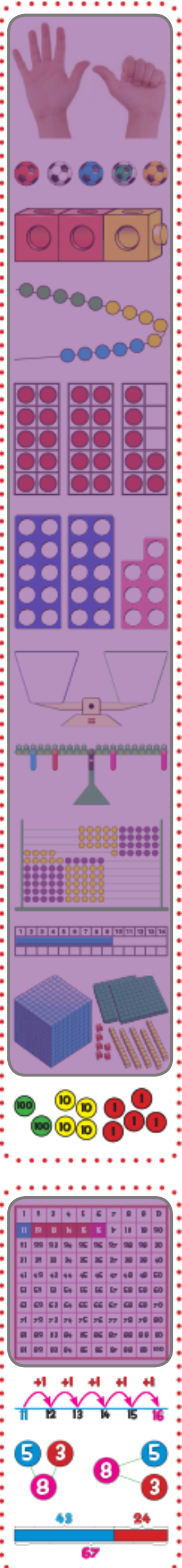
Tens Ones Tenth Hundredths

6

10 1 $\frac{1}{10}$ $\frac{1}{100}$

$$\begin{array}{r} 72.43 \\ - 47.85 \\ \hline 8 \end{array}$$

Tens Ones Tenth Hundredths



12.4 - 5.97 = 6.43

1

Tens Ones ■ Tenths Hundredths

10 1 ■ $\frac{1}{10}$ $\frac{1}{100}$

12.4
- 5.97

2

Tens Ones ■ Tenths Hundredths

10 1 ■ $\frac{1}{10}$ $\frac{1}{100}$

12.4
- 5.97

3

Tens Ones ■ Tenths Hundredths

10 1 ■ $\frac{1}{10}$ $\frac{1}{100}$

12.4
- 5.97

4

Tens Ones ■ Tenths Hundredths

10 1 ■ $\frac{1}{10}$ $\frac{1}{100}$

12.4
- 5.97

6.43

5

Tens Ones ■ Tenths Hundredths

10 1 ■ $\frac{1}{10}$ $\frac{1}{100}$

12.4
- 5.97

6.43

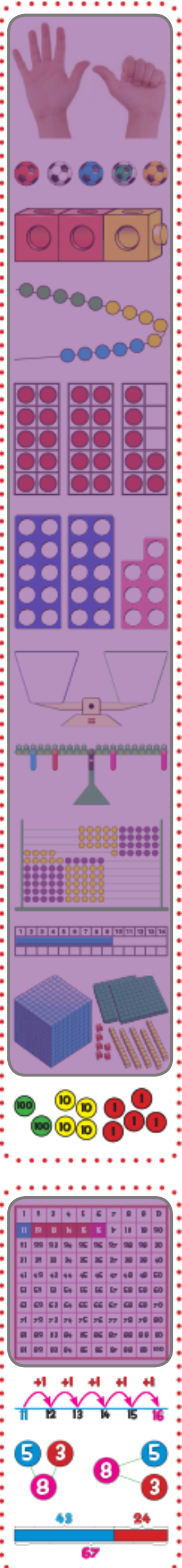
6

Tens Ones ■ Tenths Hundredths

10 1 ■ $\frac{1}{10}$ $\frac{1}{100}$

12.4
- 5.97

6.43



$$12.4 - 5.97 = 6.43$$

7

Tens Ones Tenths Hundredths

10 1 $\frac{1}{10}$ $\frac{1}{100}$

12.4
 $- 5.97$

 6.43

10 1

0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1

10

Tens Ones Tenths Hundredths

10 1 $\frac{1}{10}$ $\frac{1}{100}$

12.4
 $- 5.97$

 6.43

1 1 1 1 1 1 1 1 1 1

0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1

8

Tens Ones Tenths Hundredths

10 1 $\frac{1}{10}$ $\frac{1}{100}$

12.4
 $- 5.97$

 6.43

10 1

0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1

11

Tens Ones Tenths Hundredths

10 1 $\frac{1}{10}$ $\frac{1}{100}$

12.4
 $- 5.97$

 6.43

1 1 1 1 1 1 1 1 1 1

0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1

9

Tens Ones Tenths Hundredths

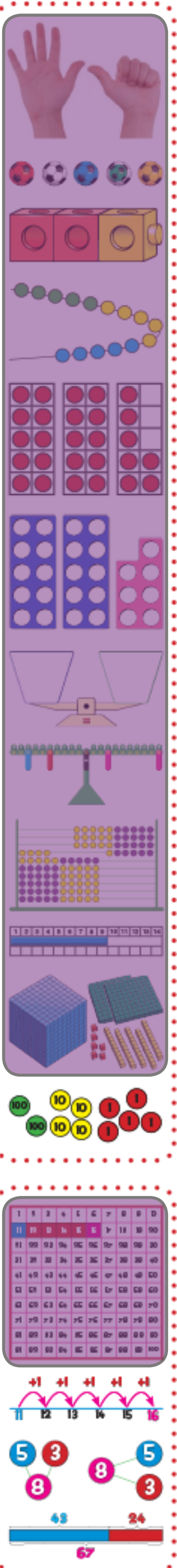
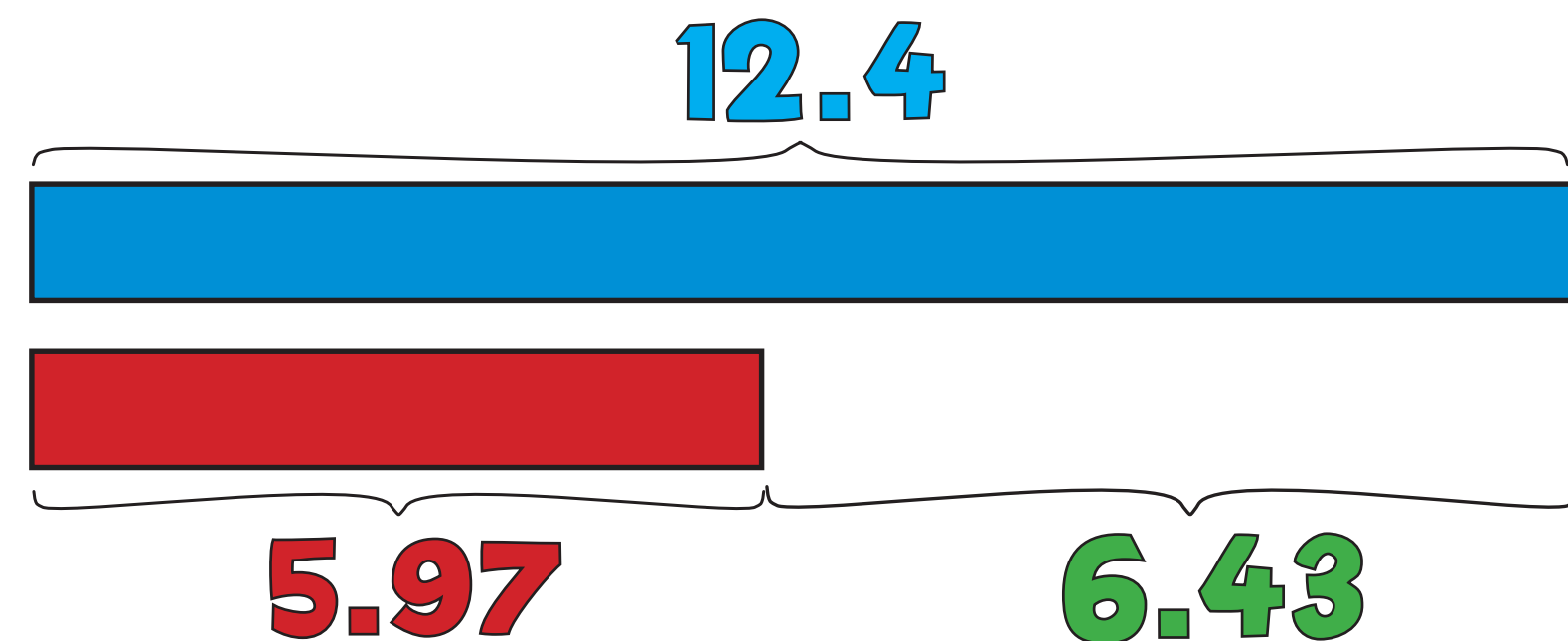
10 1 $\frac{1}{10}$ $\frac{1}{100}$

12.4
 $- 5.97$

 6.43

1 1 1 1 1 1 1 1 1 1

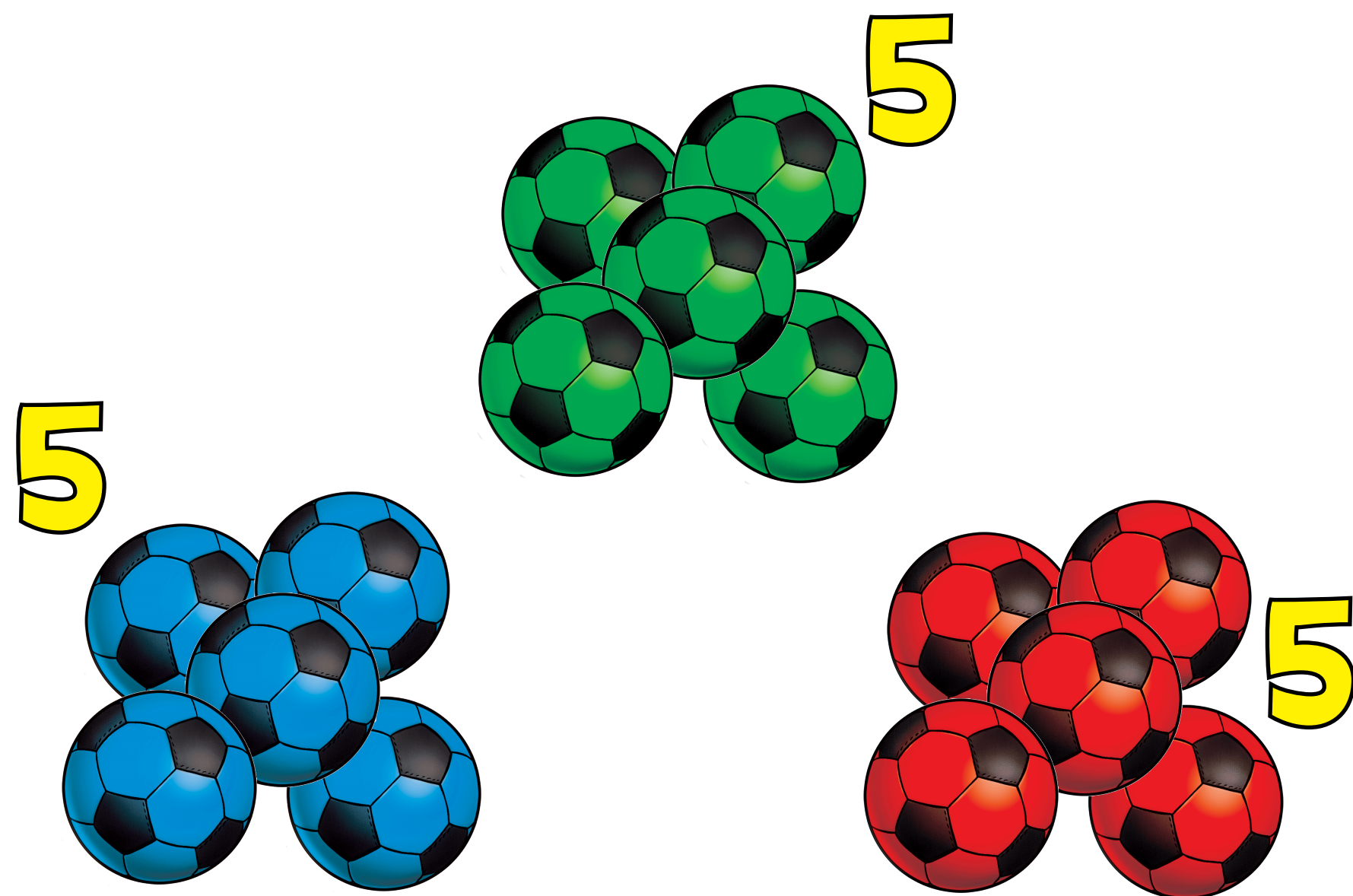
0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1



CPA Models of Multiplication

reasoning

Repeated Addition



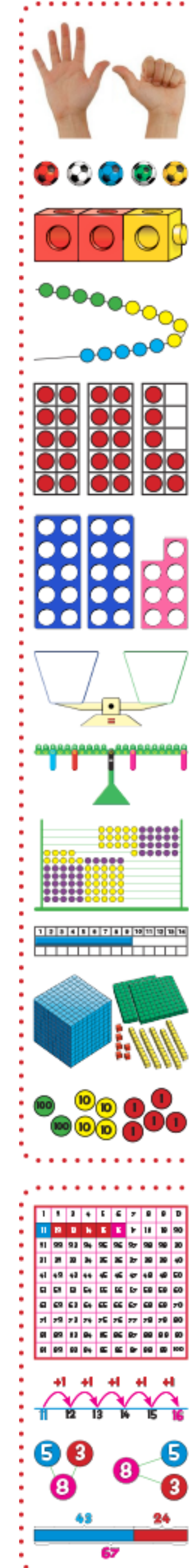
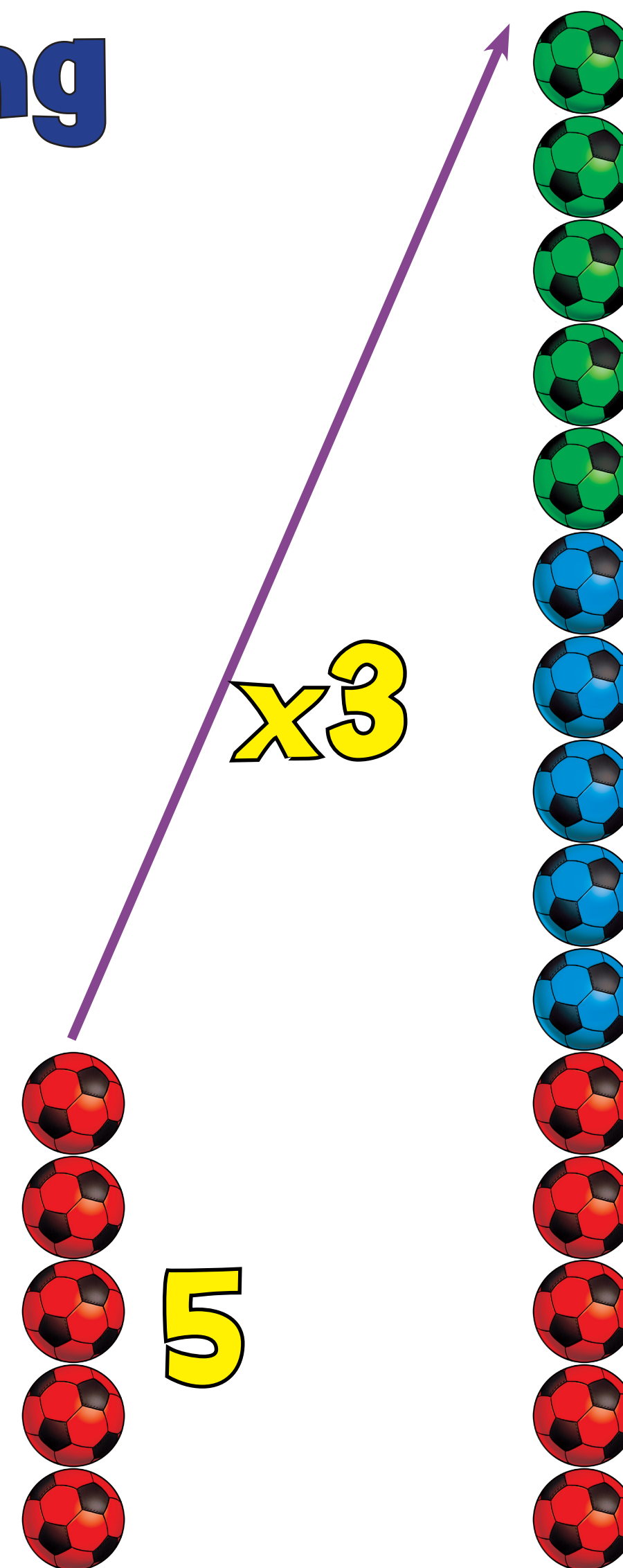
$$5 + 5 + 5 = 15$$

“I can fit 5 footballs in a bag and I’ve brought 3 full bags. How many footballs do I have with me?” “15”

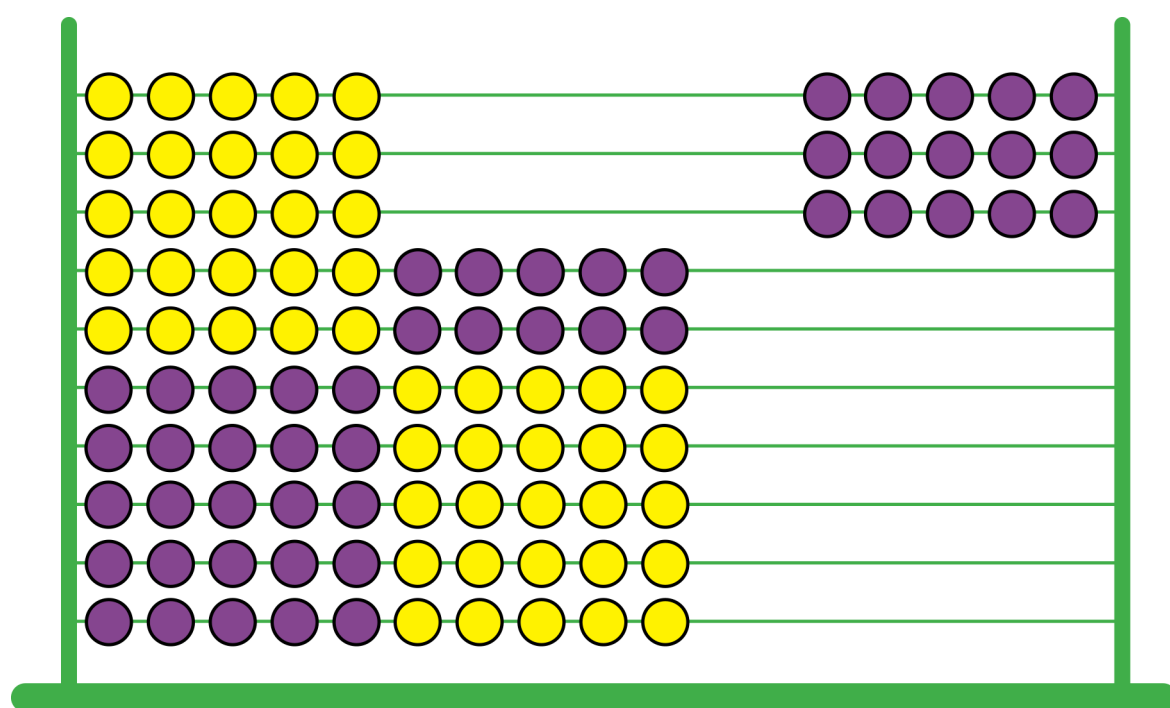
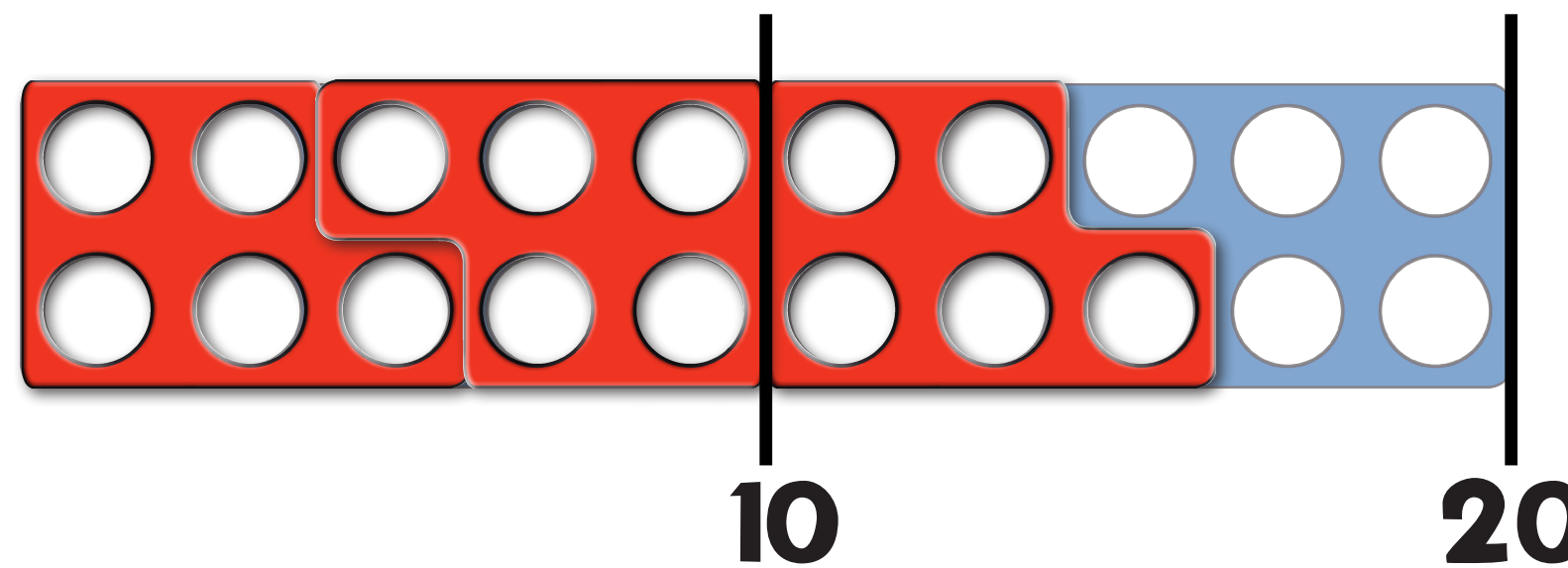
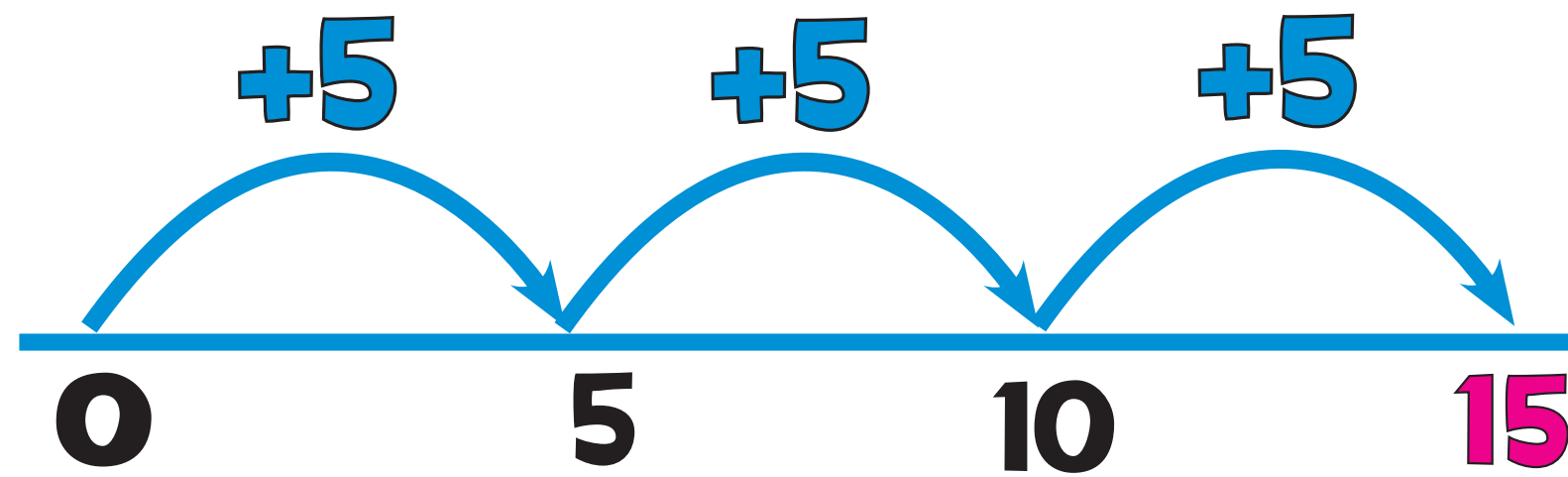
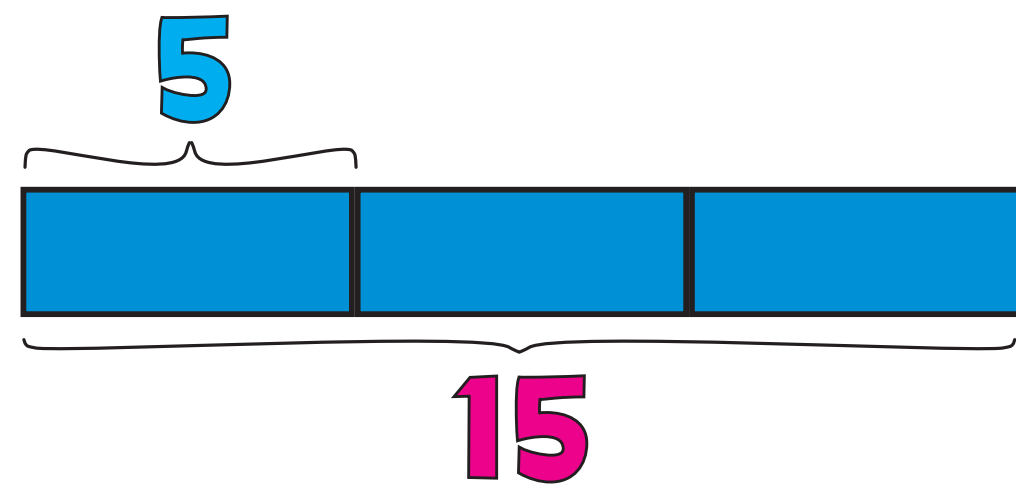
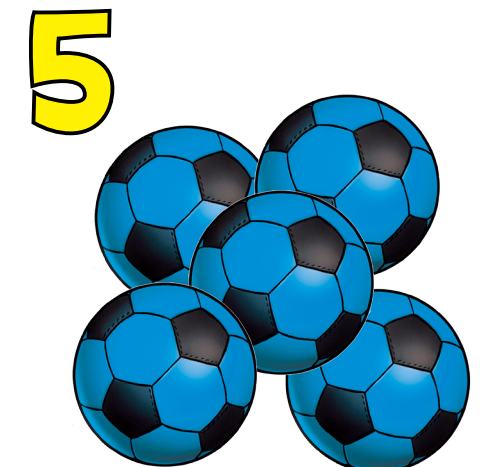
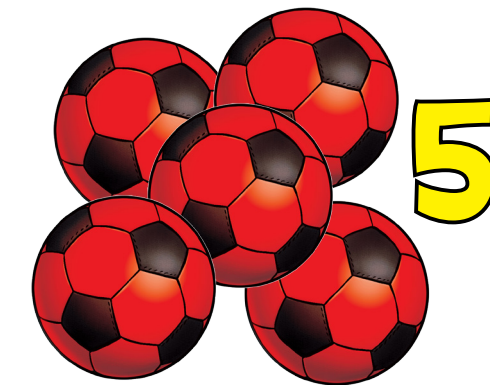
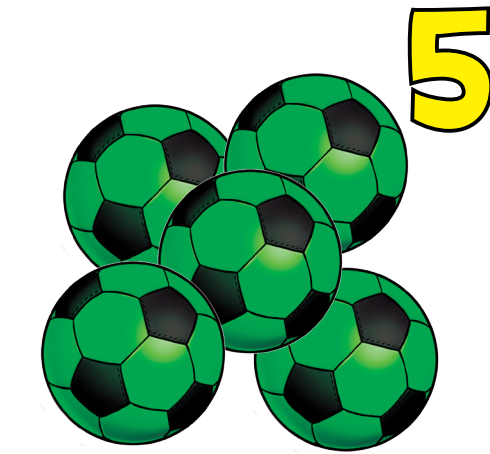
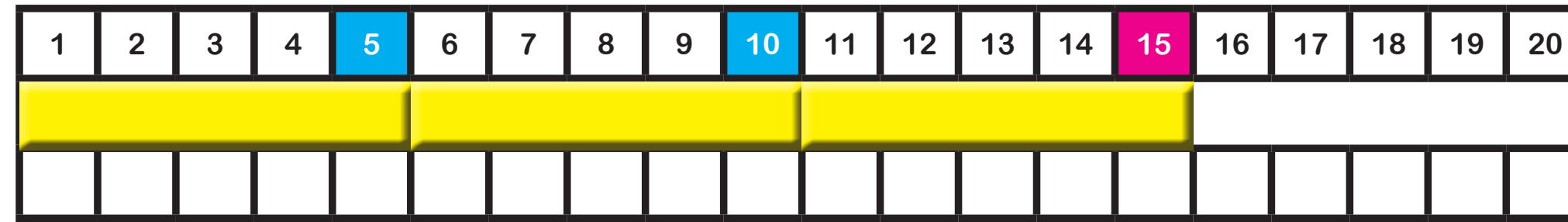
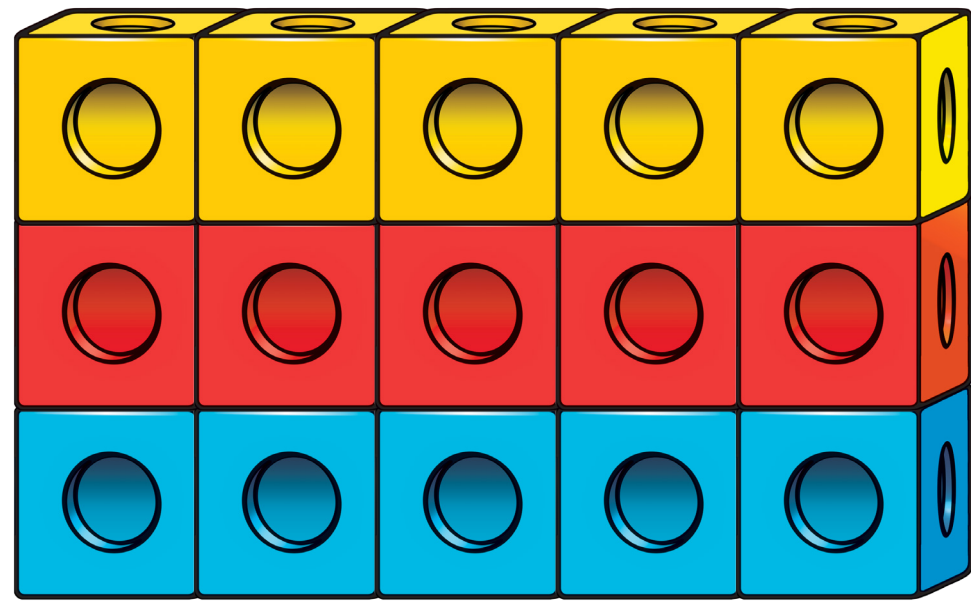
Scaling

“My tower of football stickers was 5 high, but now it’s 3 times as big! How many stickers in my tower?” “15”

$$5 \times 3 = 15$$

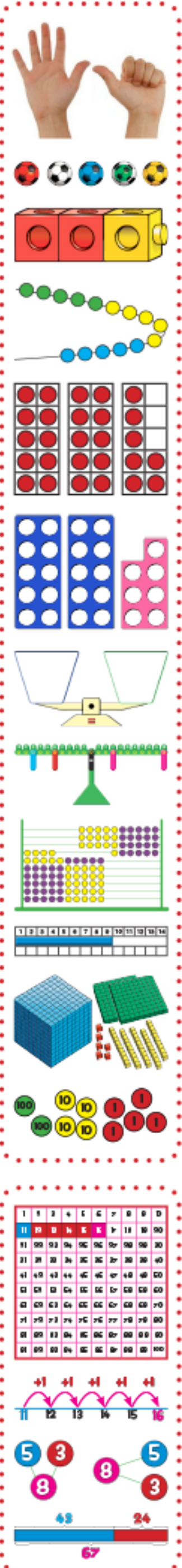


$$5 \times 3 = 15$$

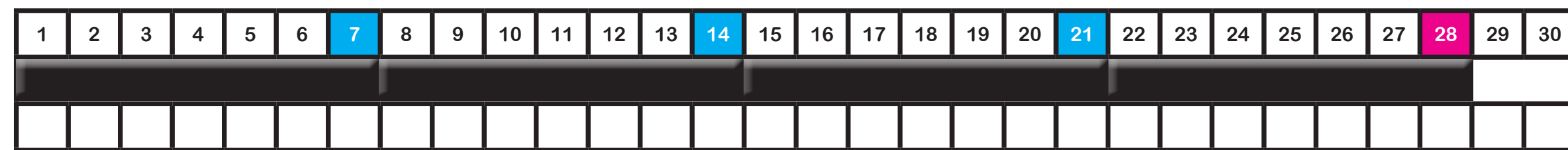
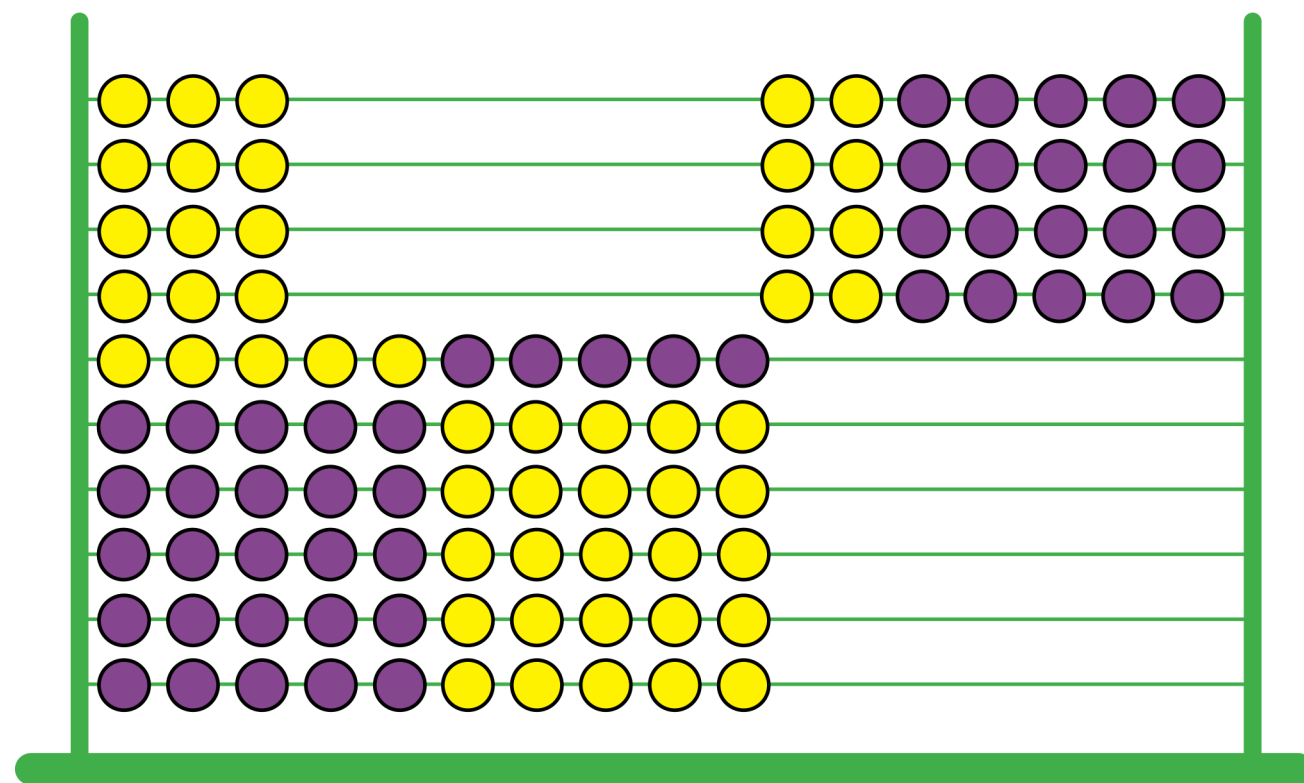
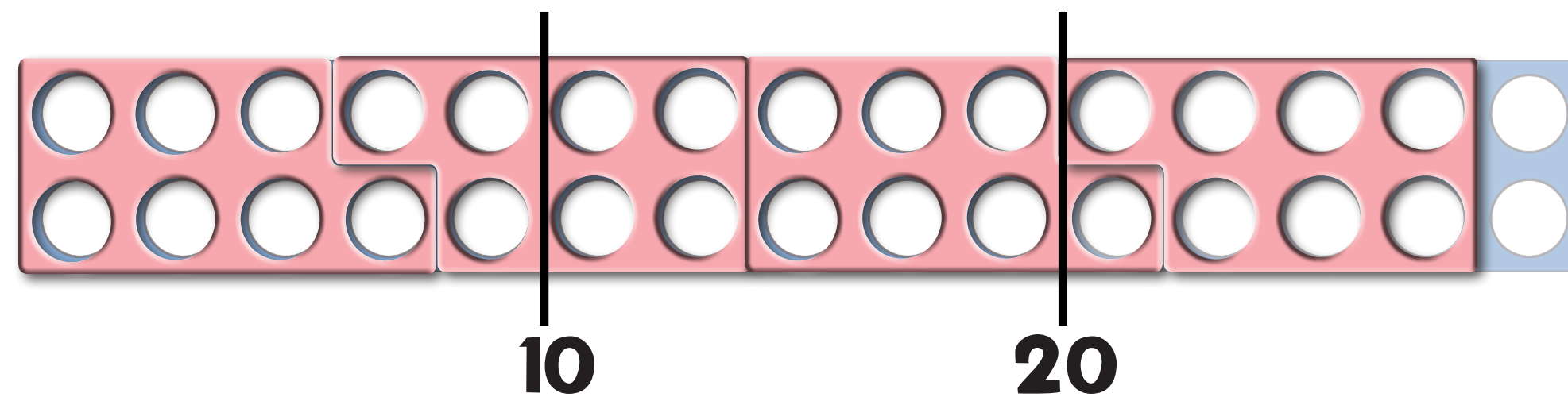
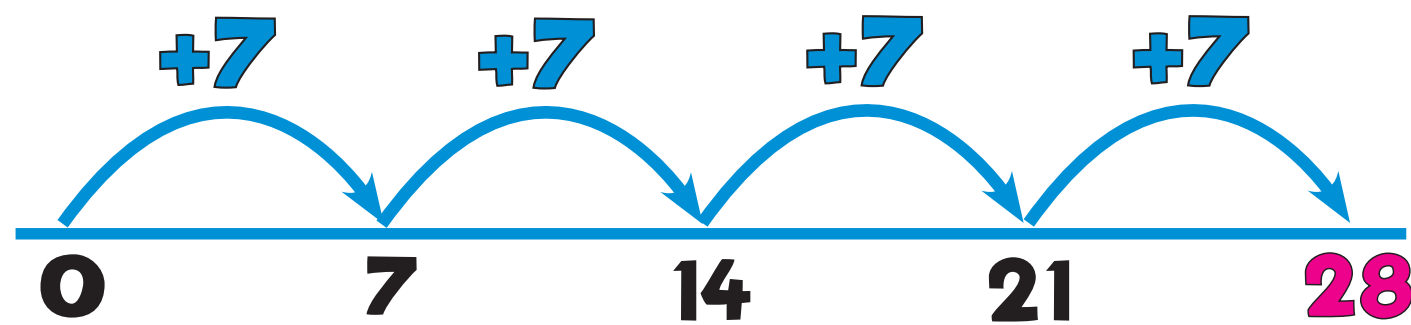
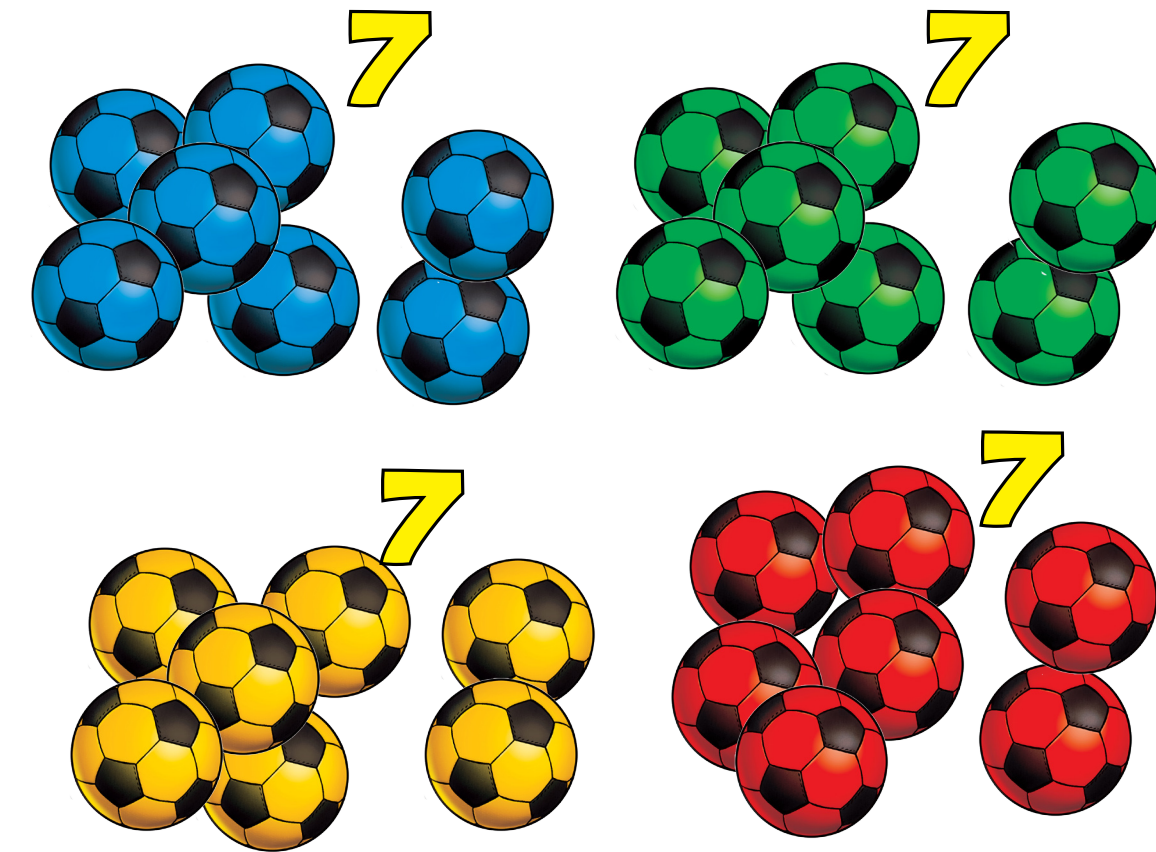
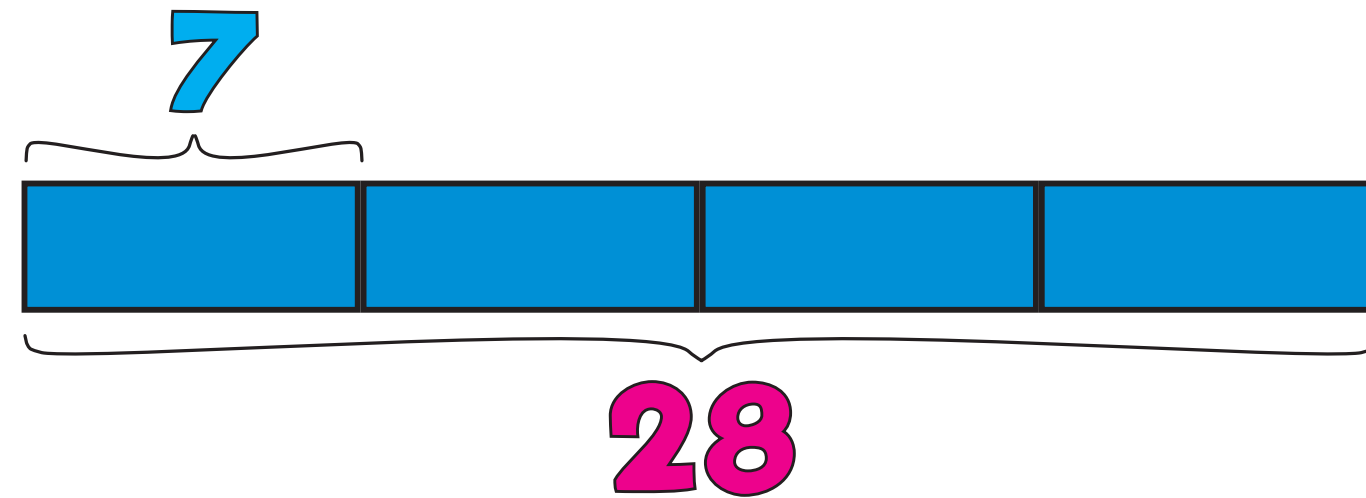
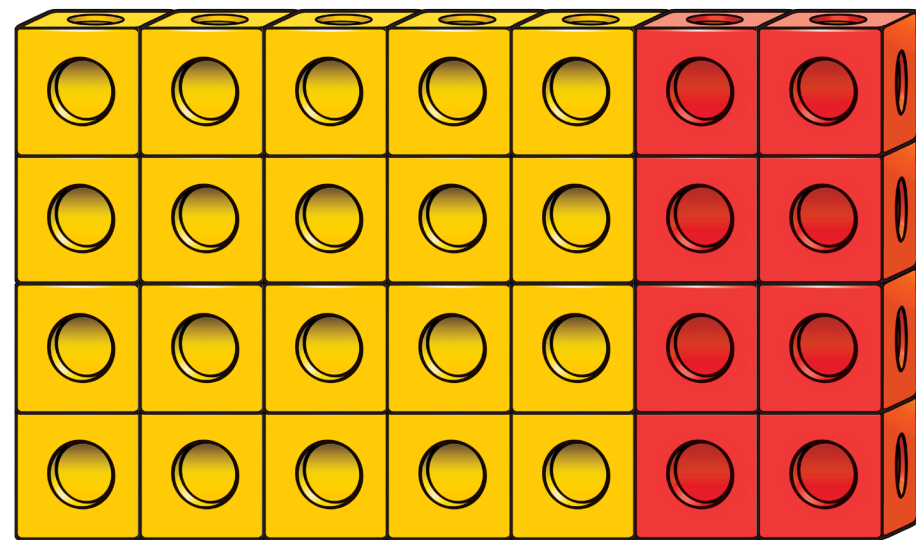


$$5 \times 3 = 5 + 5 + 5 = 15$$

“5 times 3”
“5 multiplied by 3”

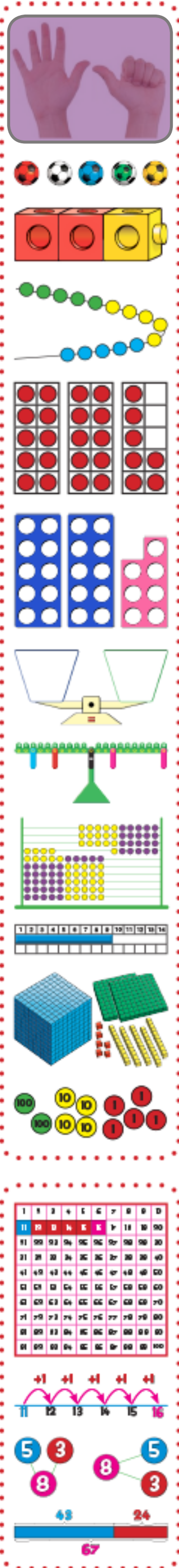


$$7 \times 4 = 28$$

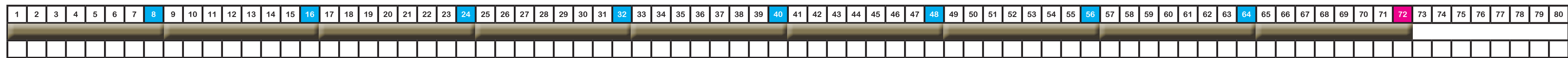
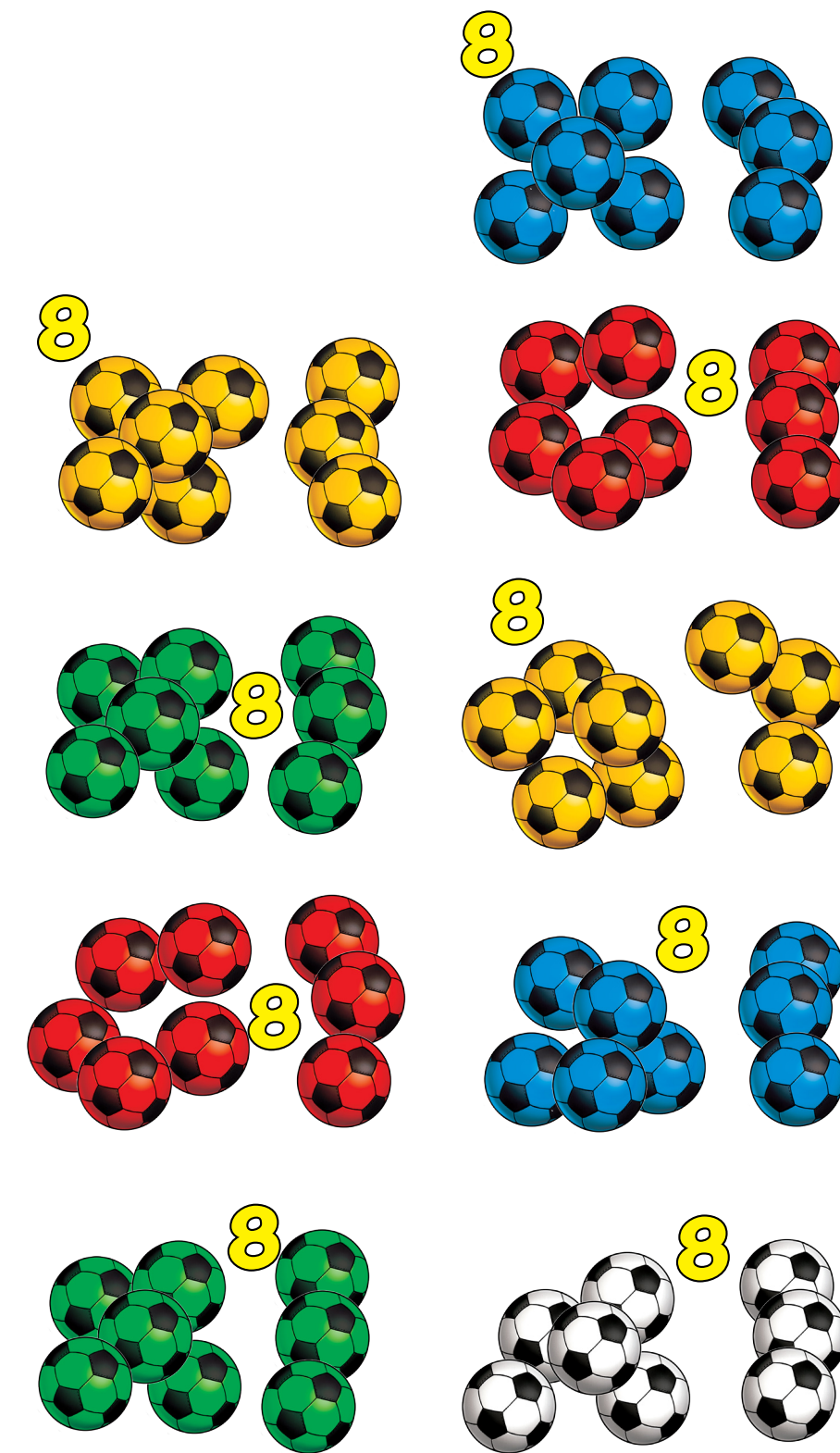
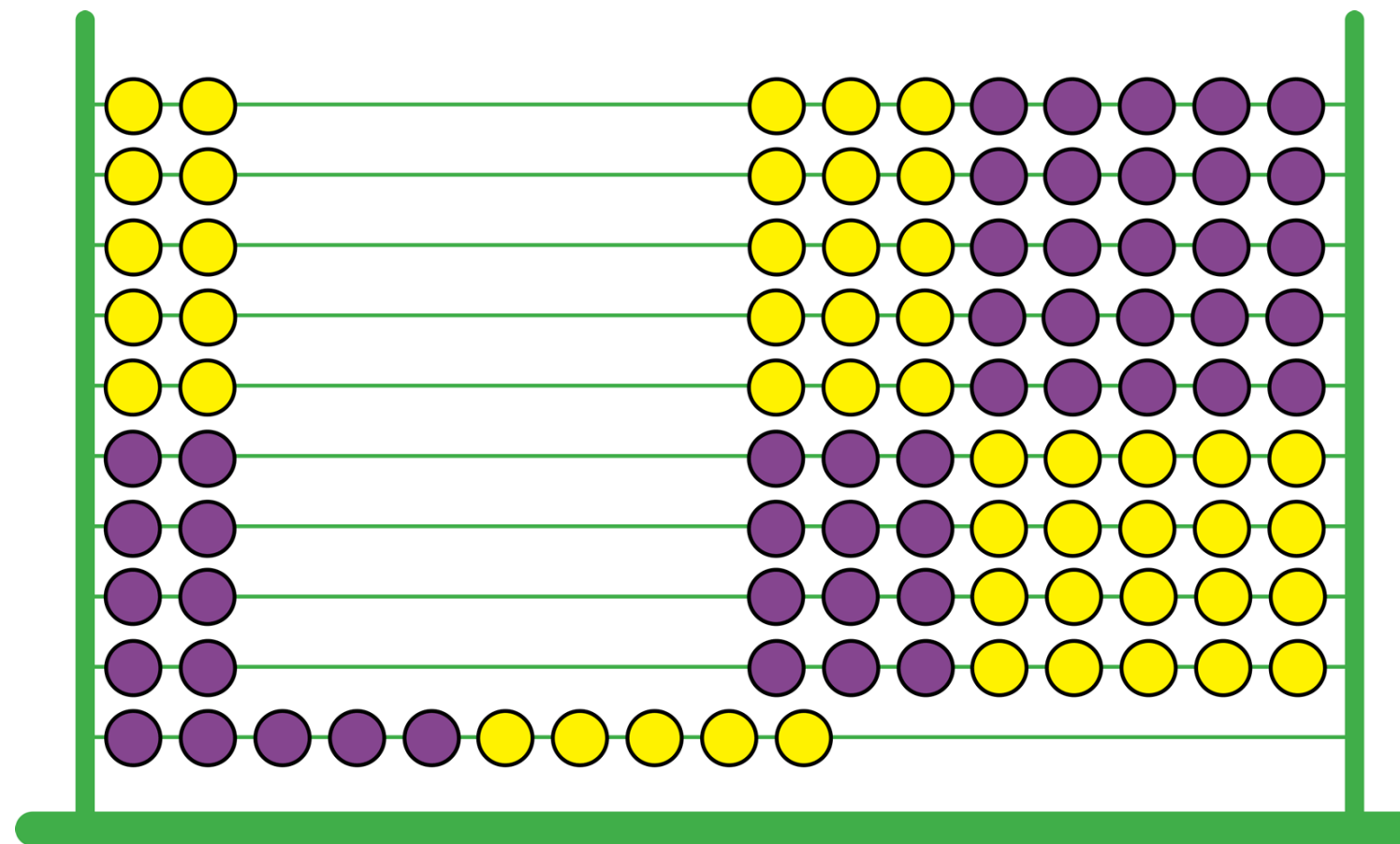
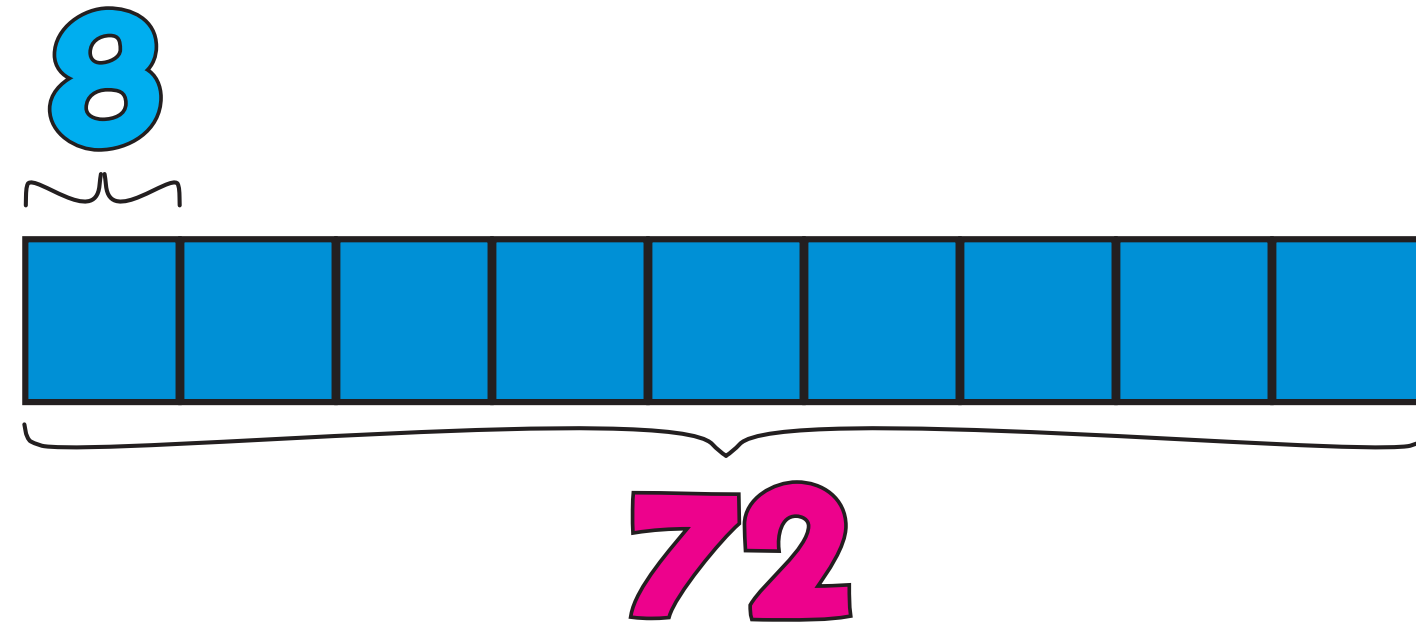
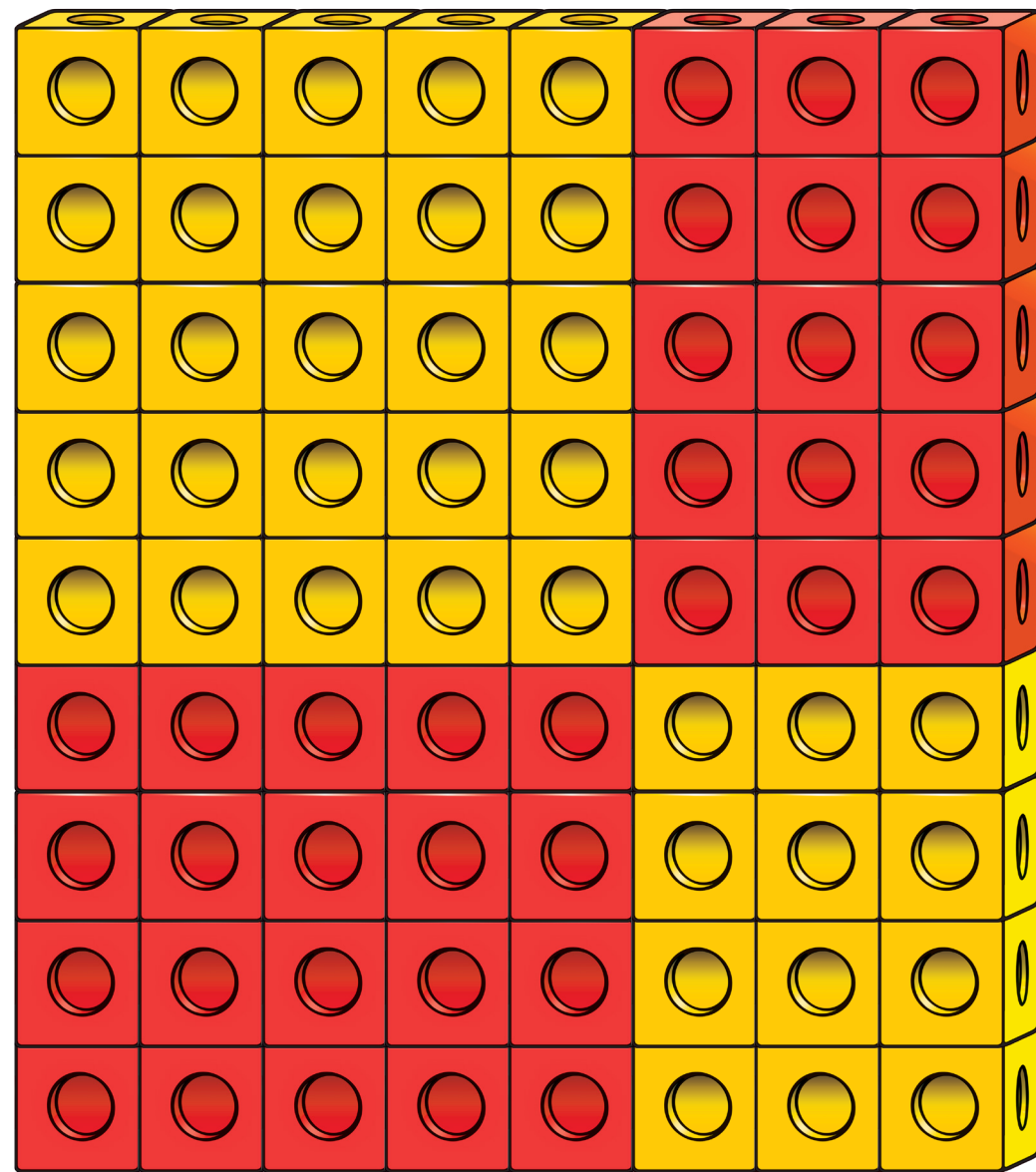


$$7 \times 4 = 7 + 7 + 7 + 7 = 28$$

“7 times 4”
“7 multiplied by 4”



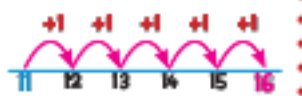
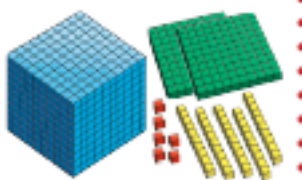
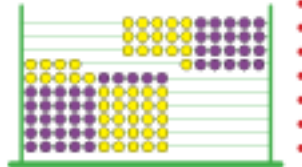
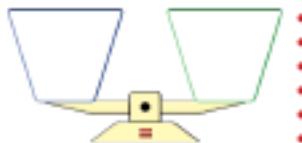
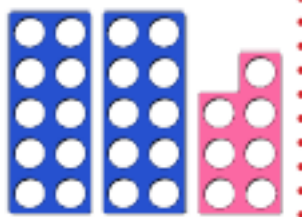
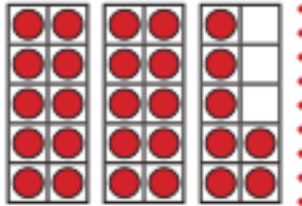
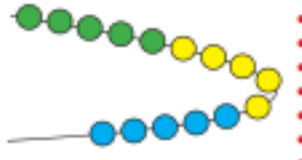
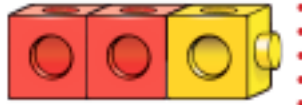
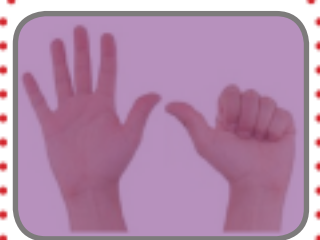
$$8 \times 9 = 72$$



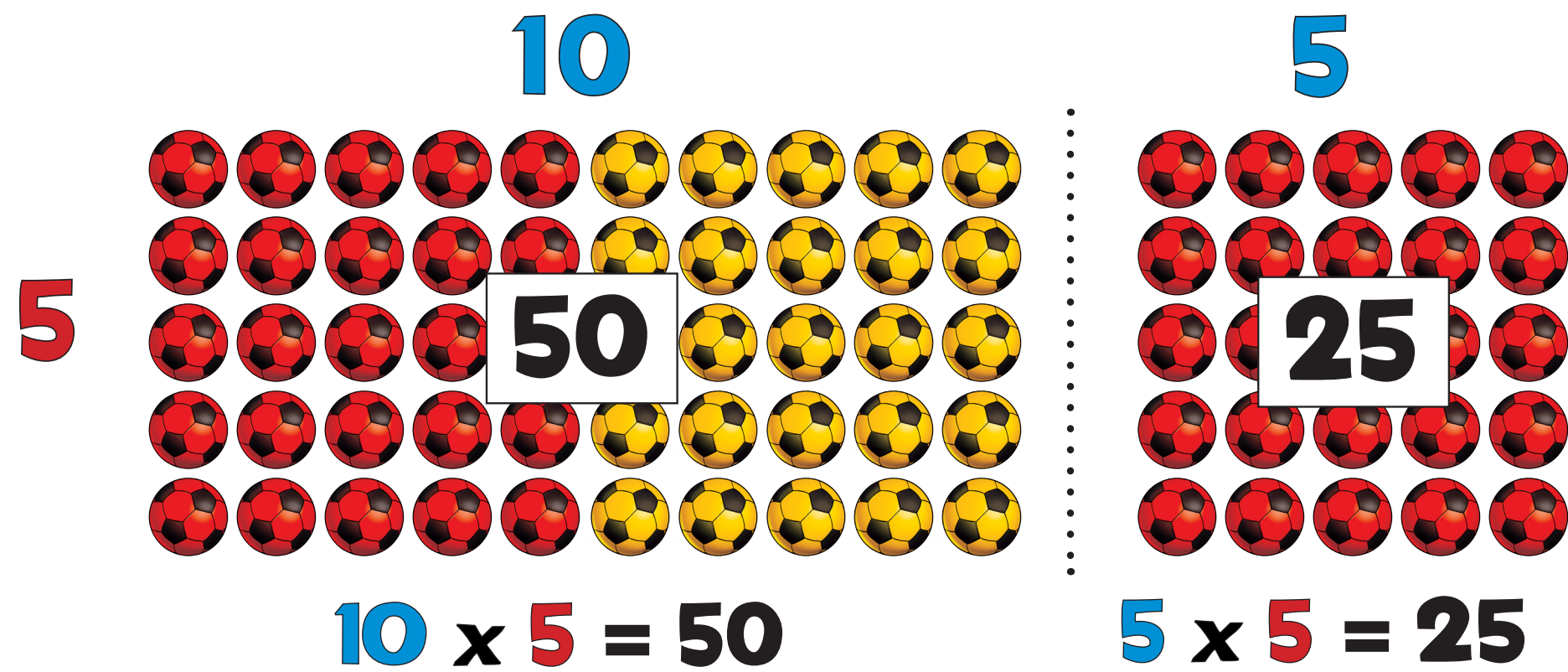
“8 multiplied by 9”

“8 times 9”

$$8 \times 9 = 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 = 72$$

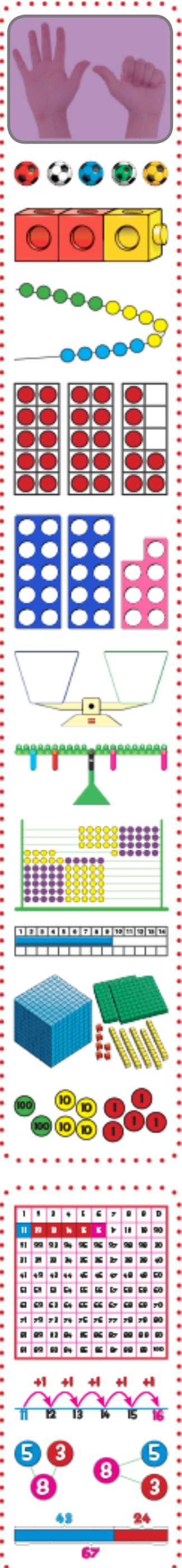
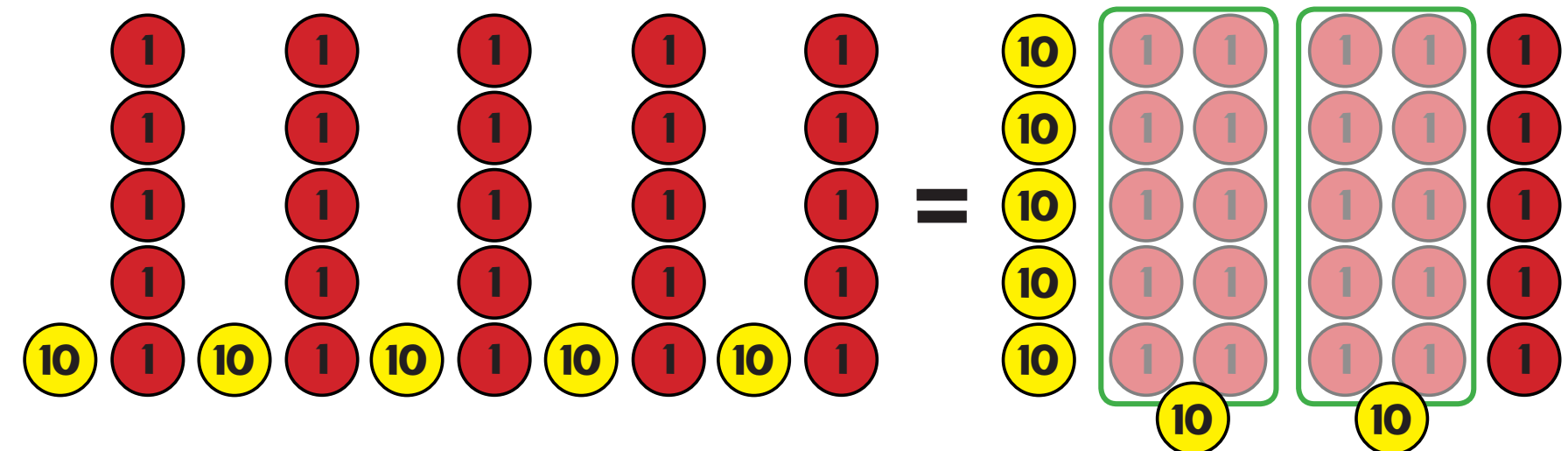
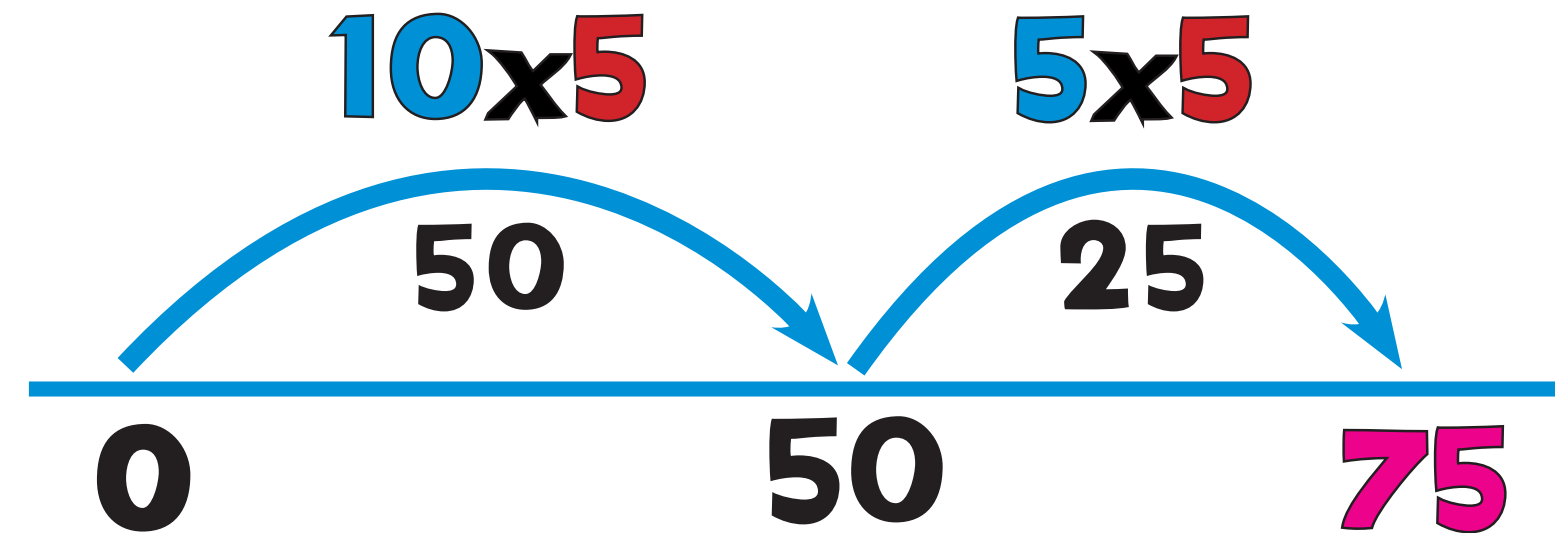
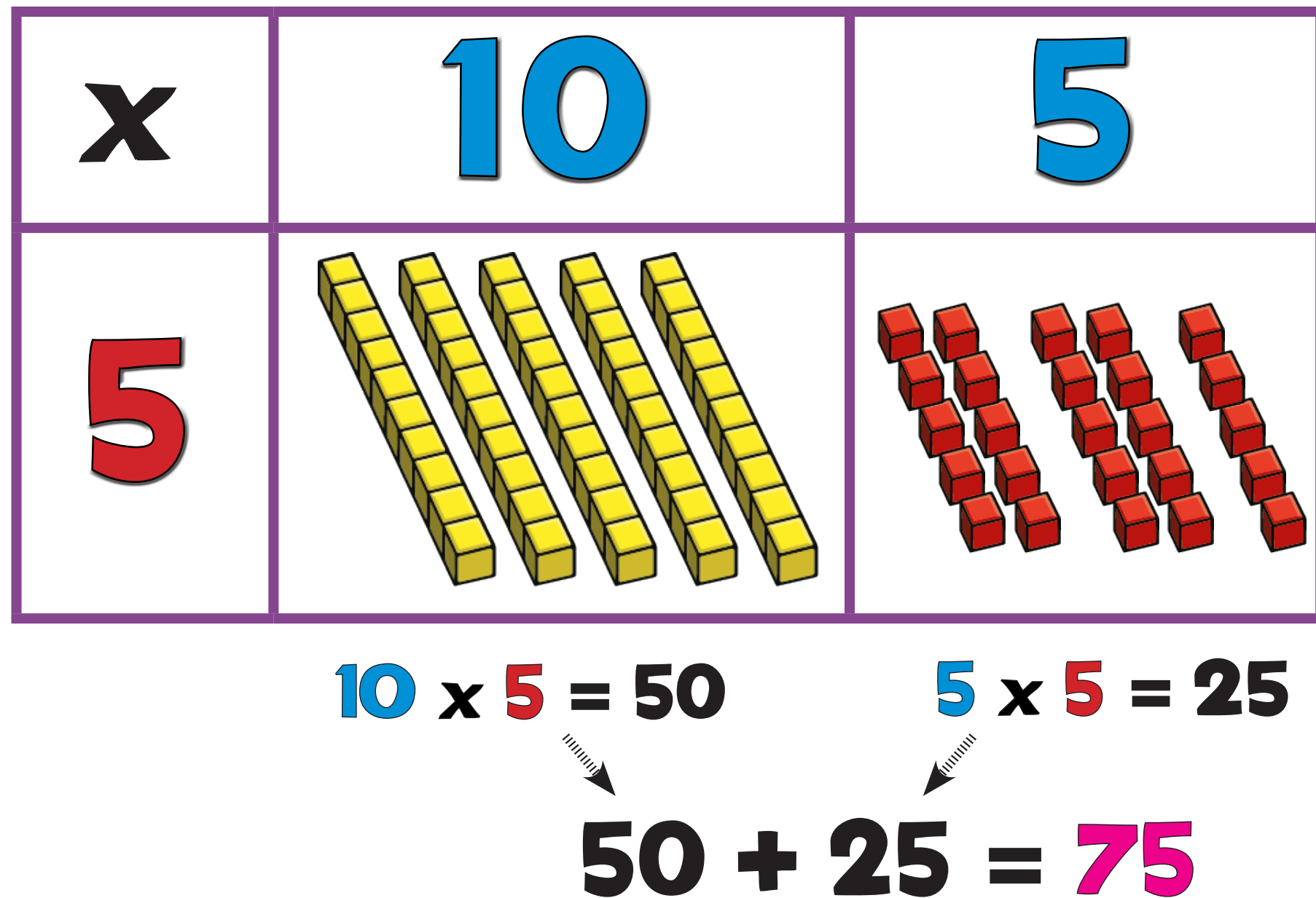


15 x 5 = 75

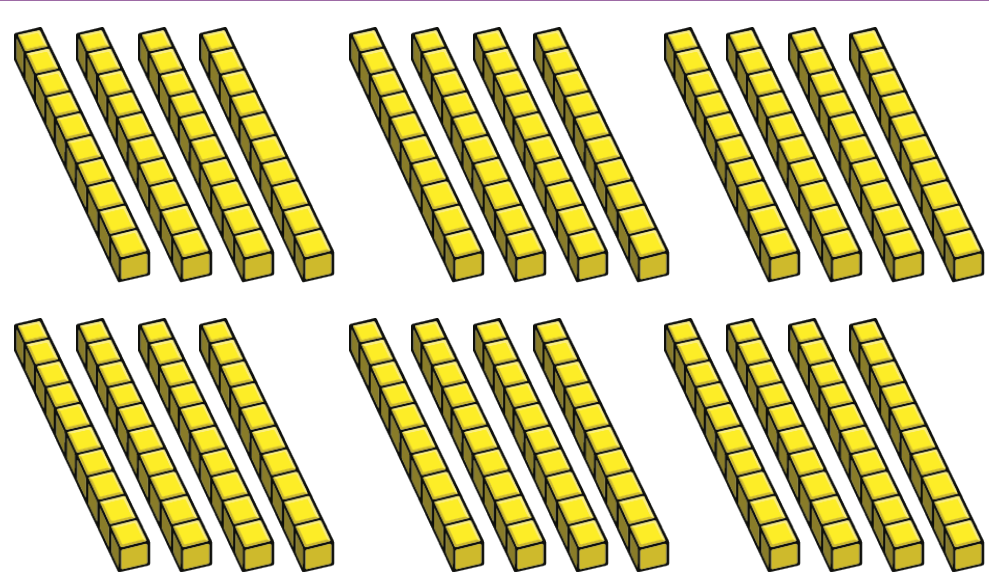
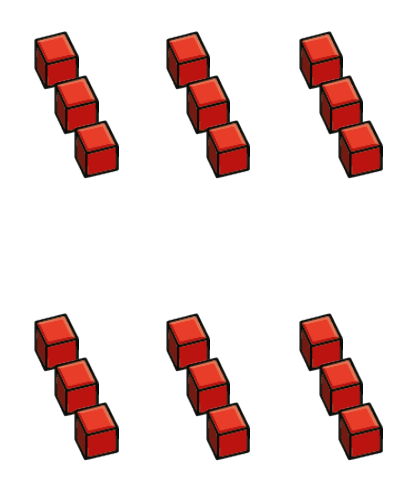


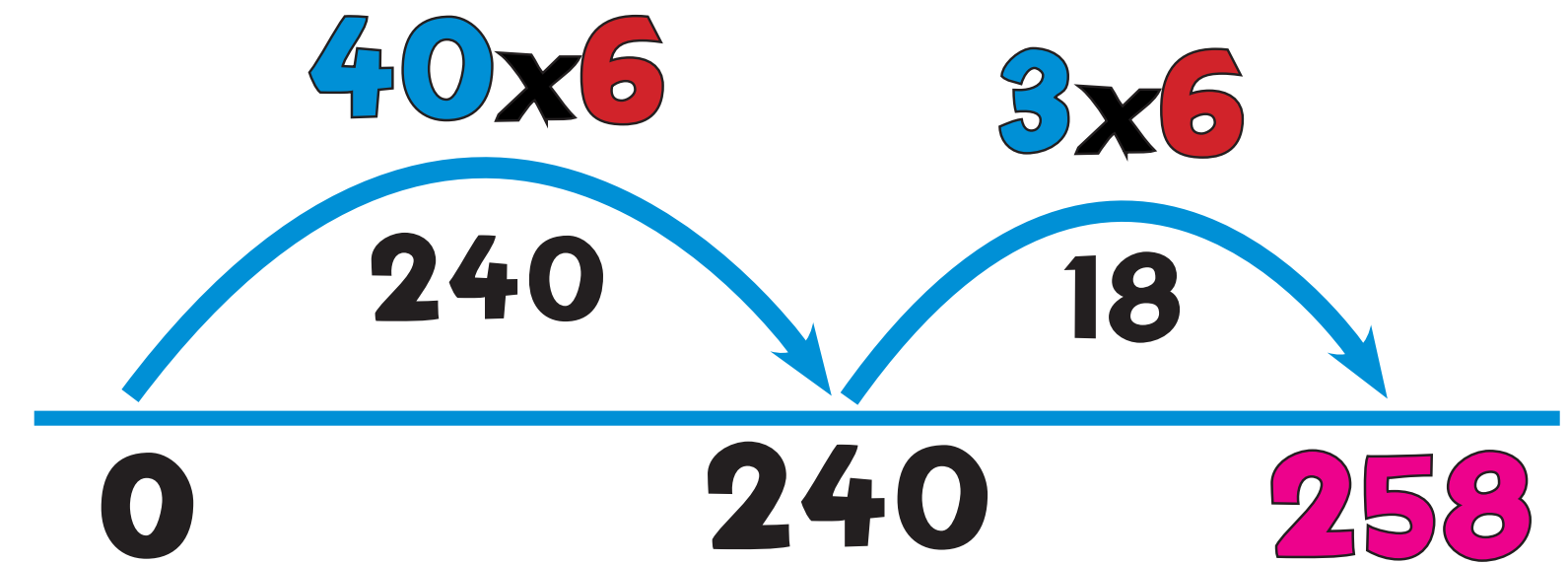
x	10	5
5	50	25

50 + 25 = 75

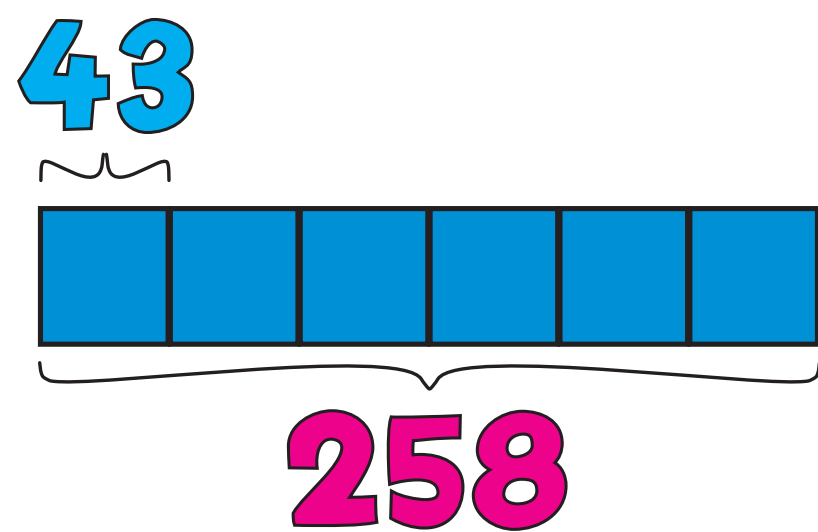


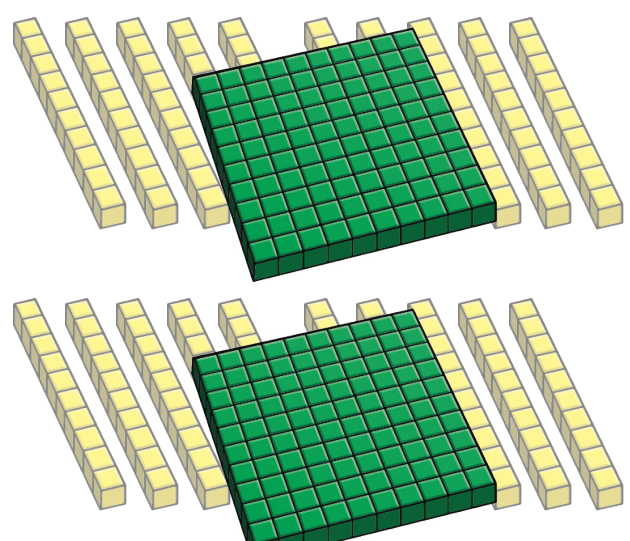
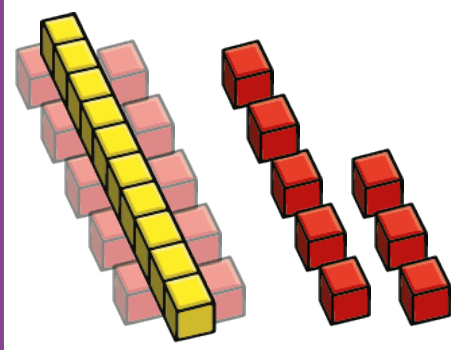
$$43 \times 6 = 258$$

x	40	3
6		

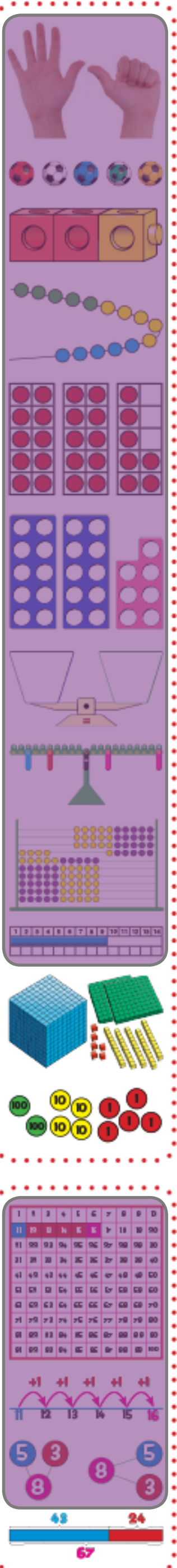


x	40	3
6	240	18

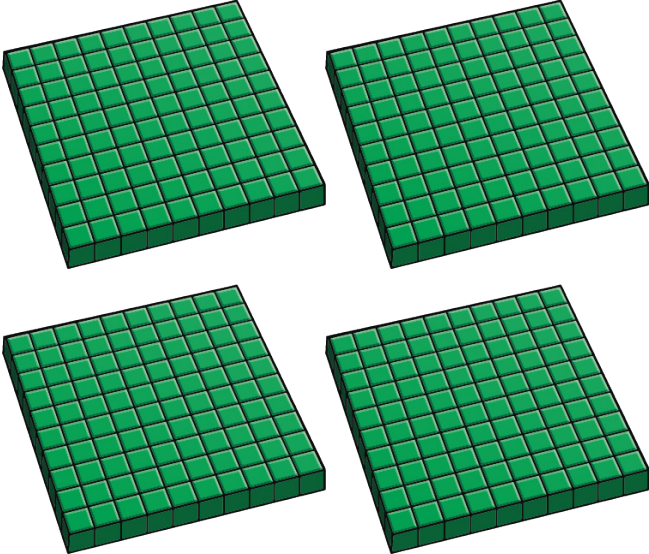
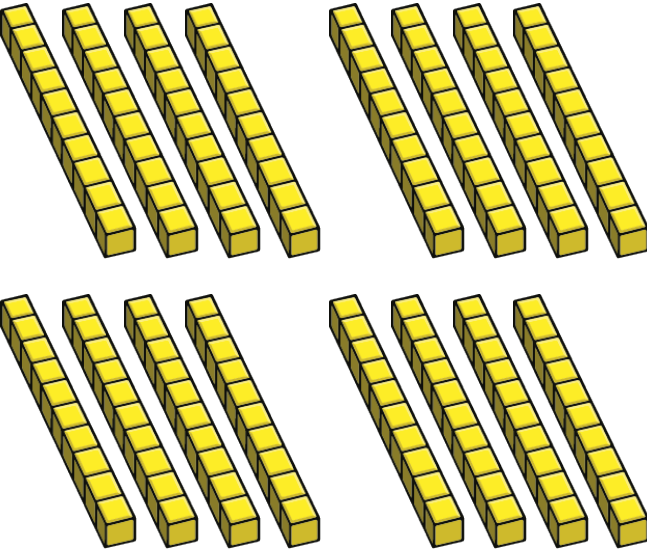
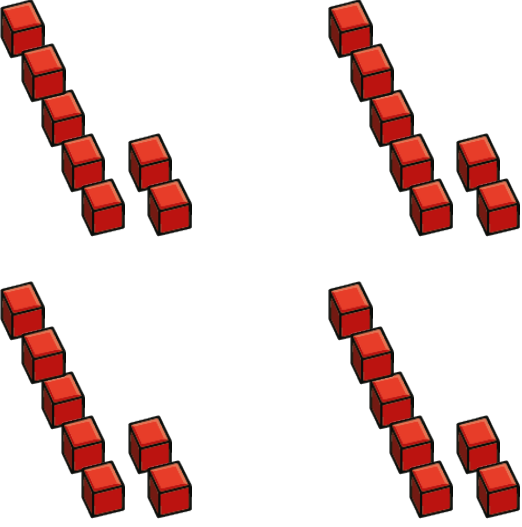


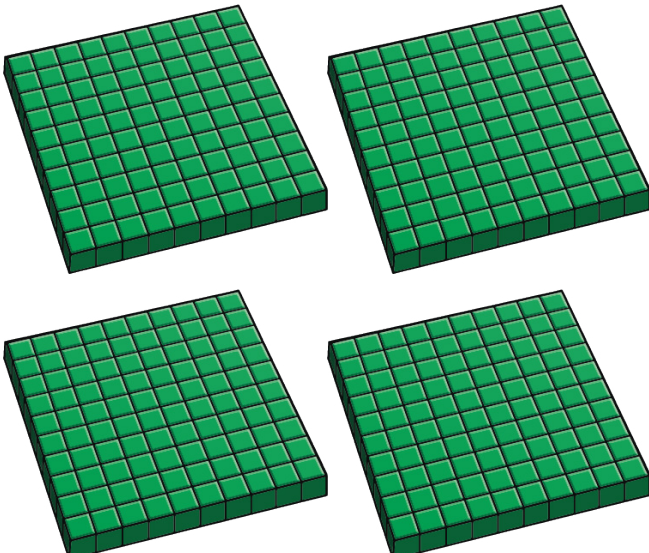
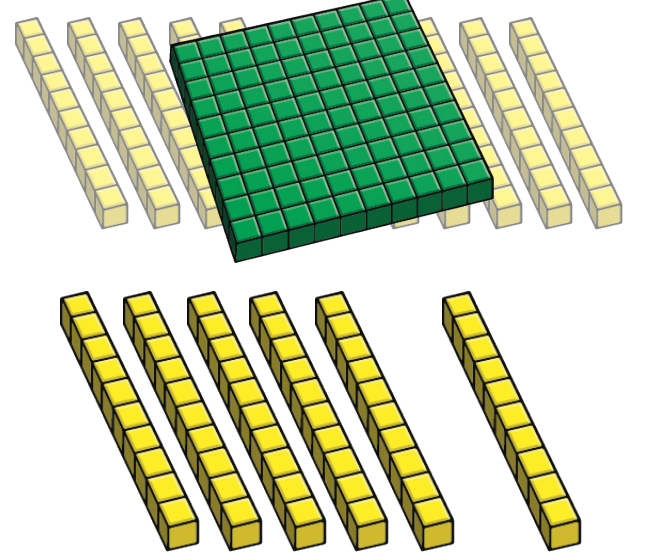
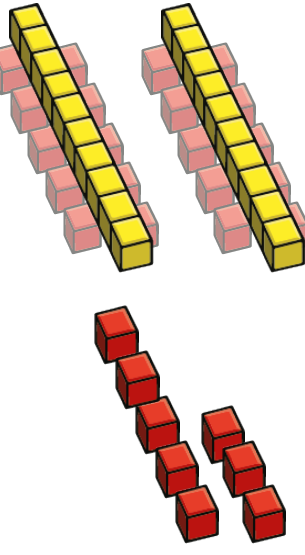
x	40	3
6		

$$240 + 18 = 258$$

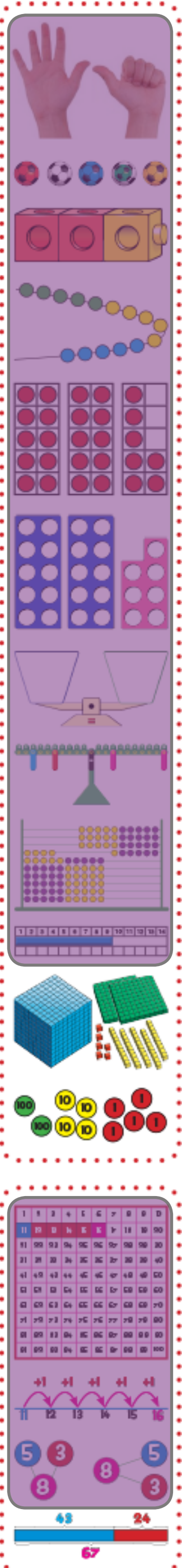


$$147 \times 4 = 588$$

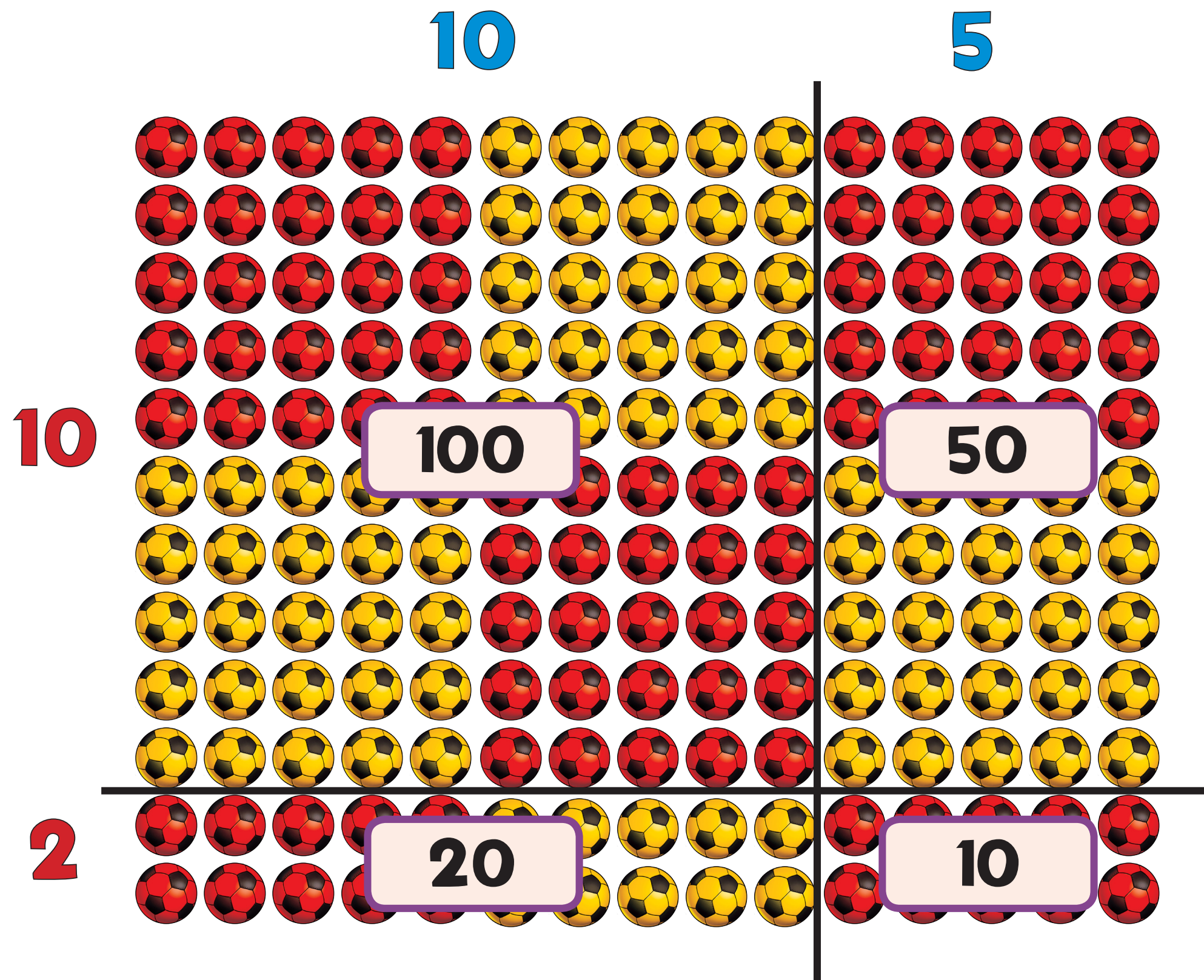
x	100	40	7
4			

x	100	40	7
4			

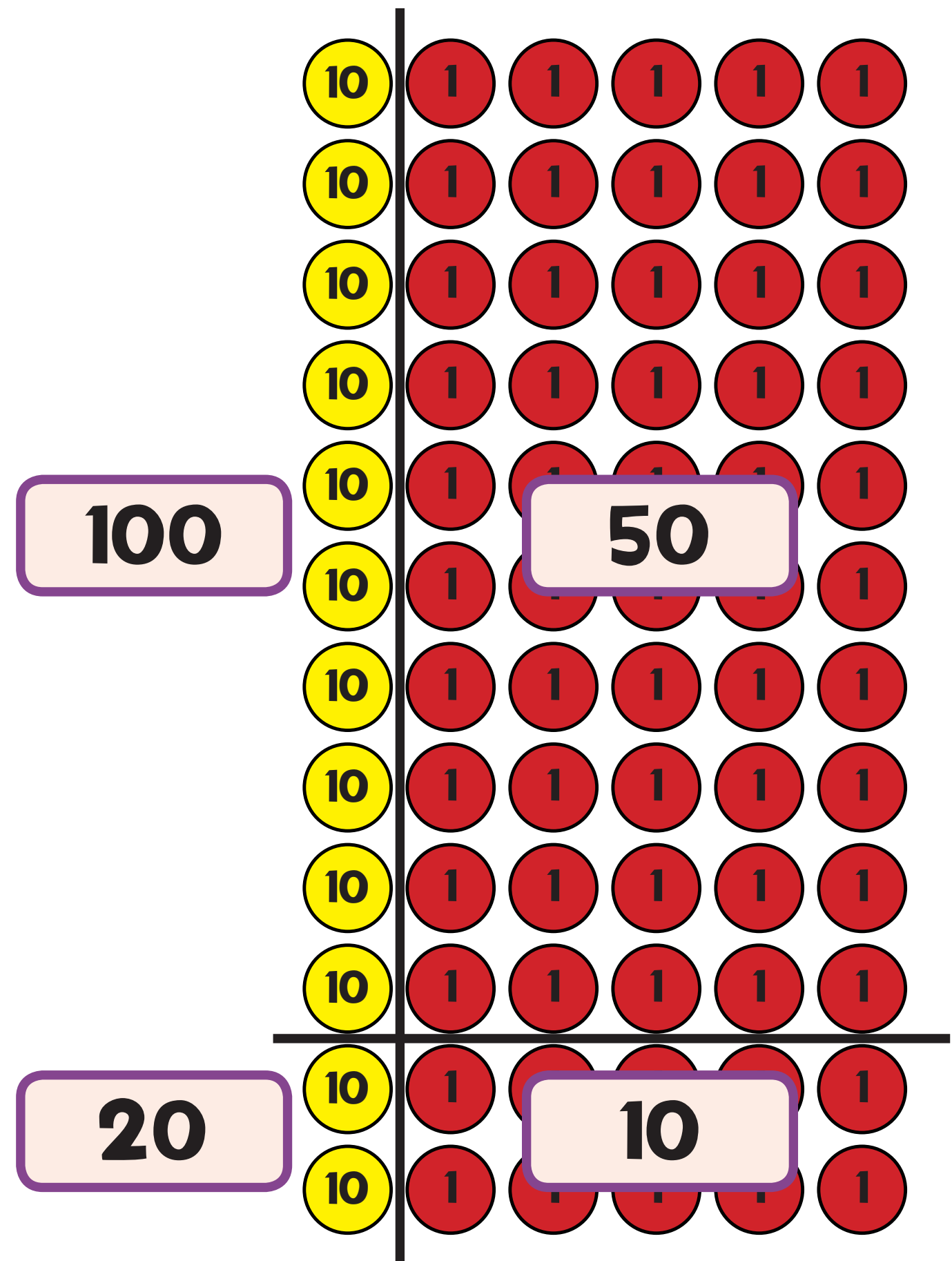
$$400 + 160 + 28 = 588$$



15 x 12 = 180

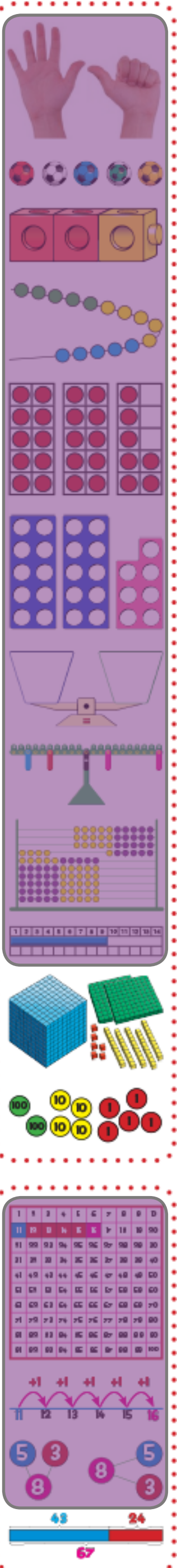


100 + 50 + 20 + 10 = 180

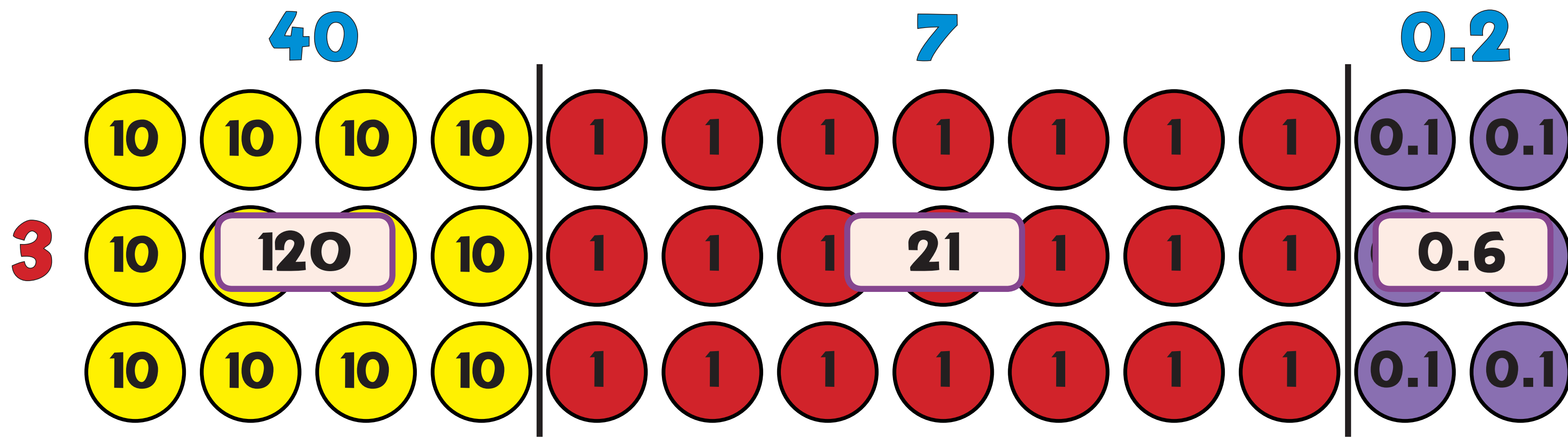


$$\begin{array}{r}
 15 \\
 \times 12 \\
 \hline
 30 \quad (15 \times 2) \\
 + 150 \quad (15 \times 10) \\
 \hline
 180
 \end{array}$$

x	10	5
10	100	50
2	20	10

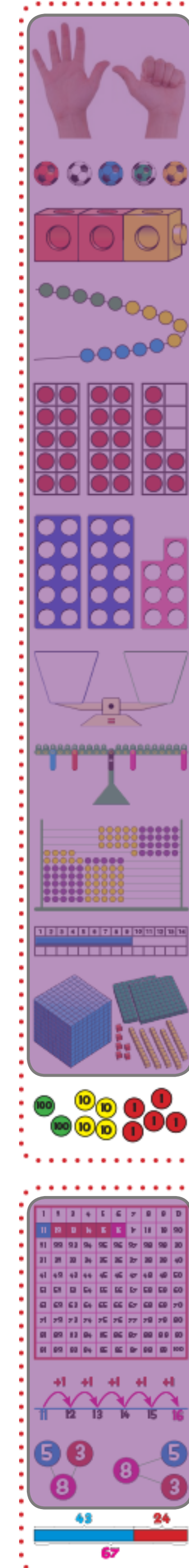


$$47.2 \times 3 = 141.6$$

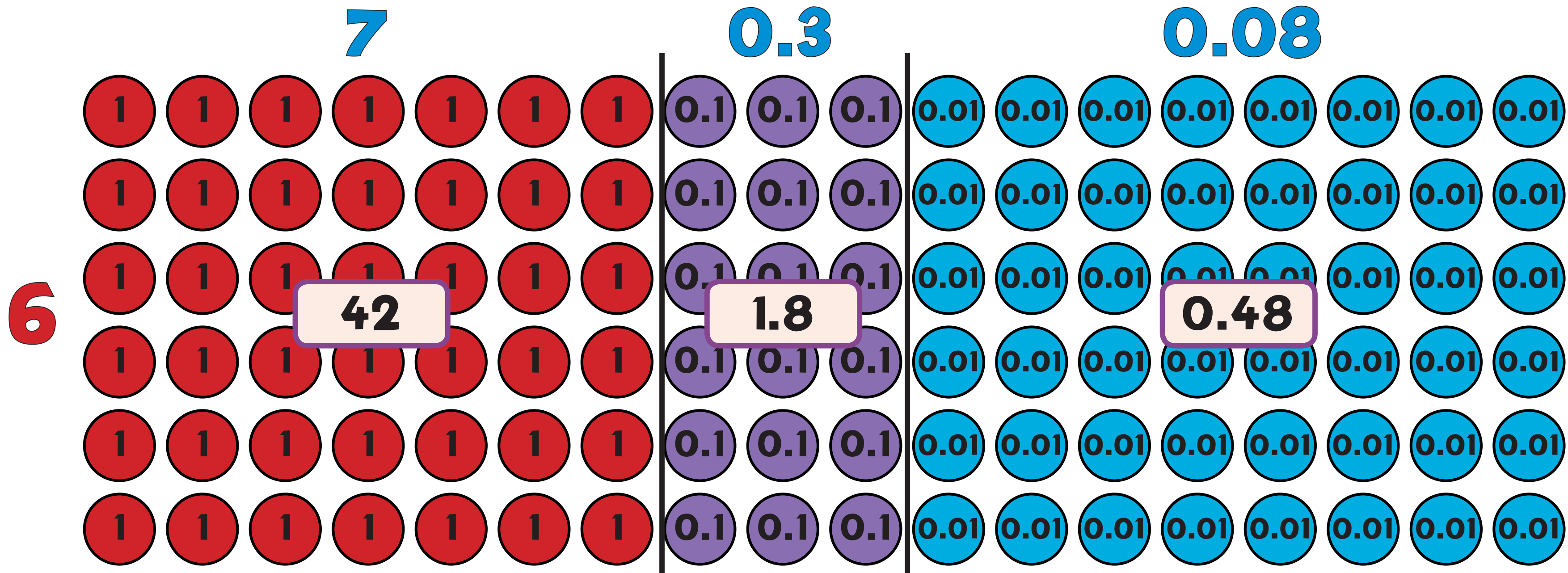


x	40	7	0.2
3	120	21	0.6

$$\begin{array}{r}
 100 \quad 10 \quad 1 \quad \frac{1}{10} \\
 47.2 \\
 \times 3 \\
 \hline
 141.6 \\
 \hline
 2
 \end{array}$$

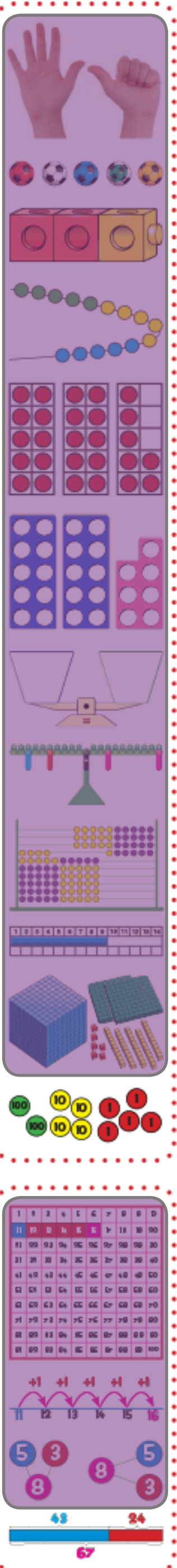


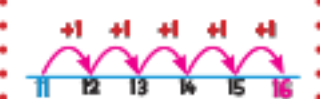
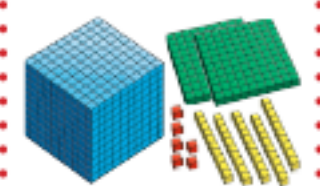
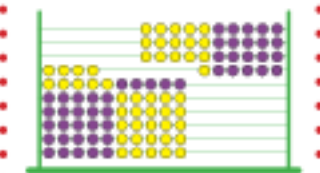
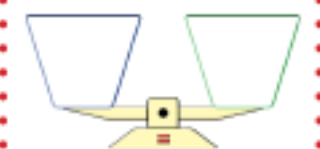
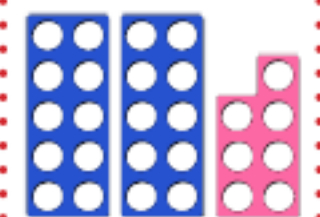
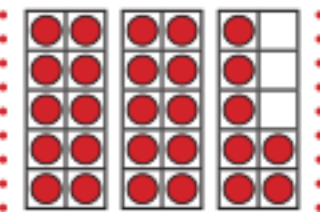
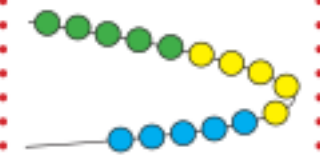
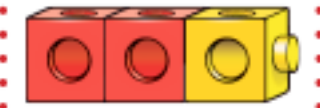
$$7.38 \times 6 = 44.28$$



x	7	0.3	0.08
6	42	1.8	0.48

$$\begin{array}{r}
 10 \quad 1 \quad \frac{1}{10} \quad \frac{1}{100} \\
 7.38 \\
 \times 6 \\
 \hline
 44.28 \\
 \hline
 4 \quad 2 \quad 4
 \end{array}$$





Sharing



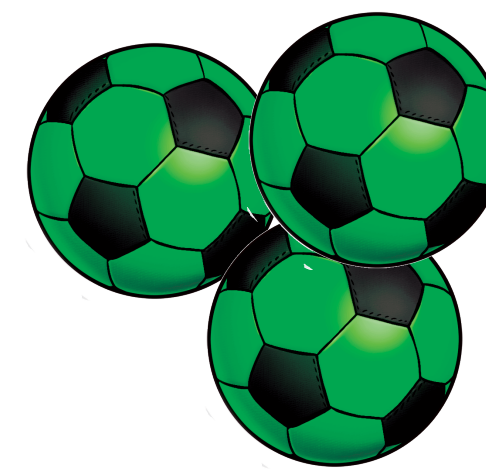
5 in each group!



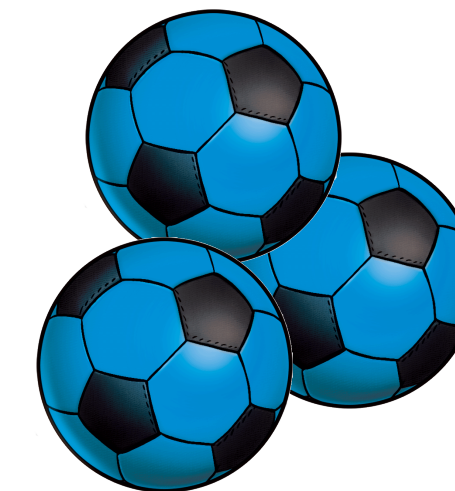
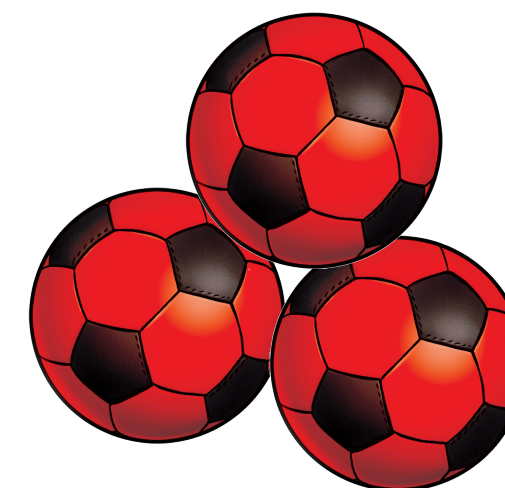
$$15 \div 3 = 5$$

"If I shared my 15 footballs fairly into 3 bags, how many balls would be in each bag?" "5"

Grouping



5 groups



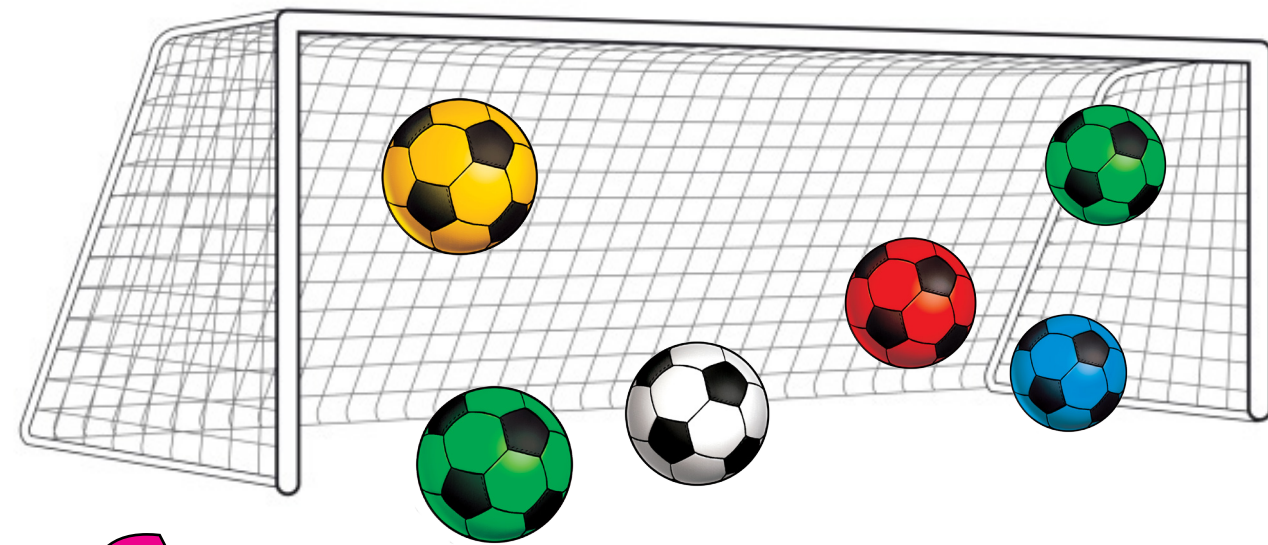
$$15 \div 3 = 5$$

"If I can put my 15 footballs into groups of 3, how many groups would I create?" "5"

$$12 \div 2 = 6$$

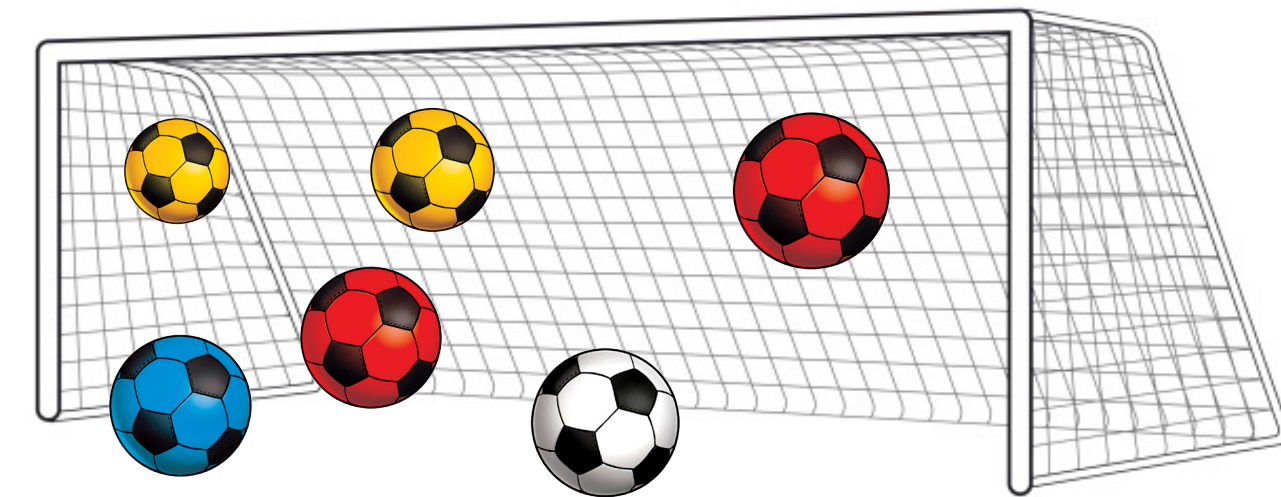


Sharing Model

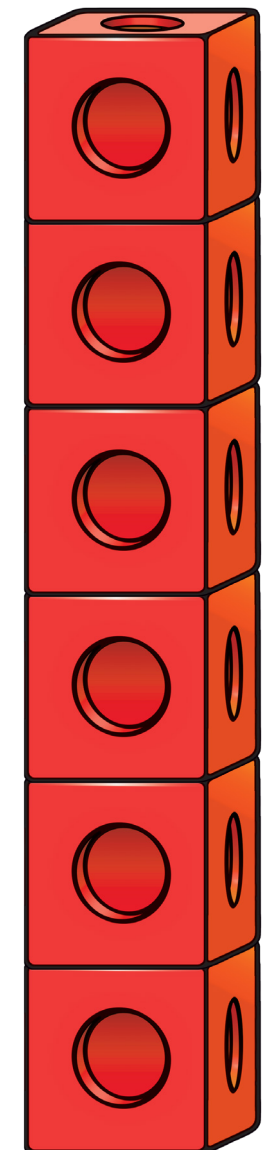


6

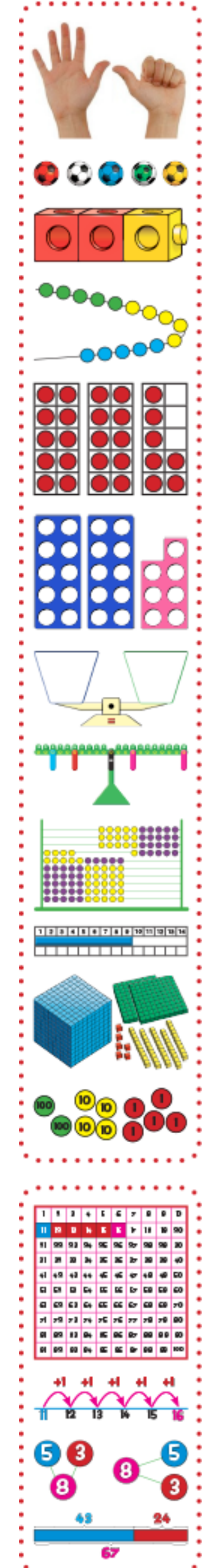
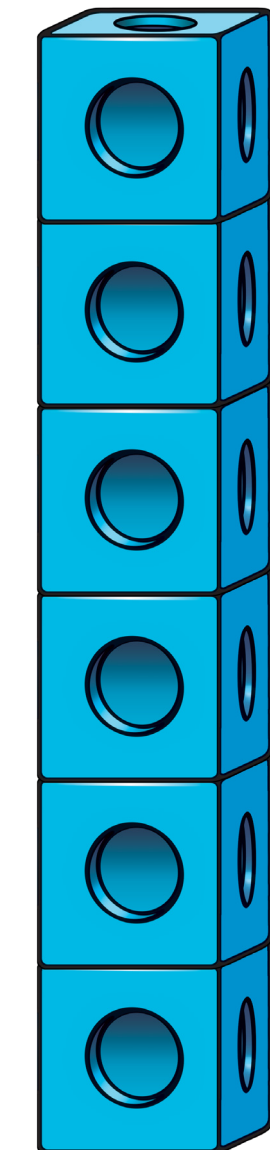
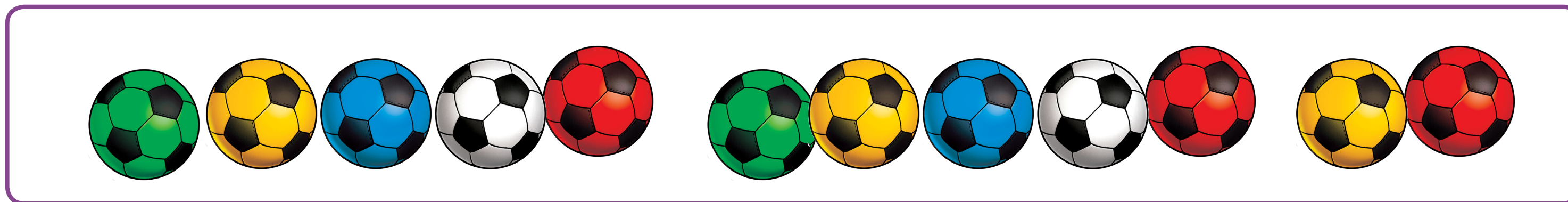
**6 footballs
in each goal!**



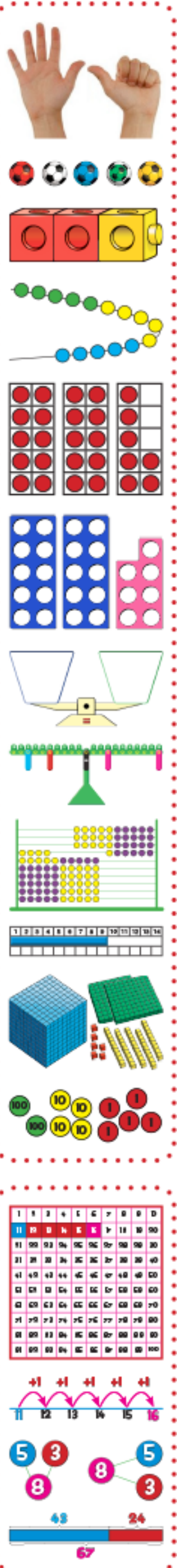
6



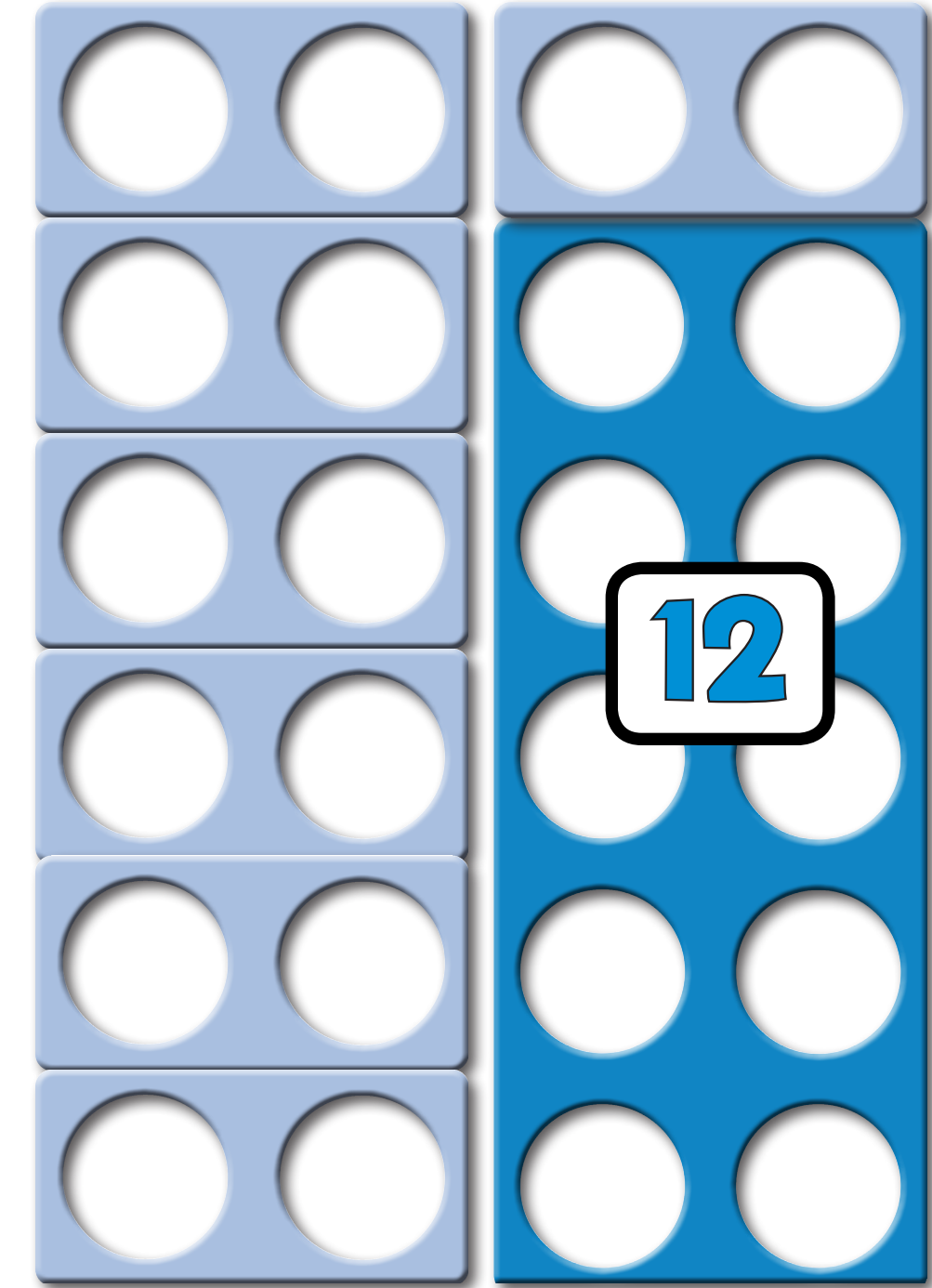
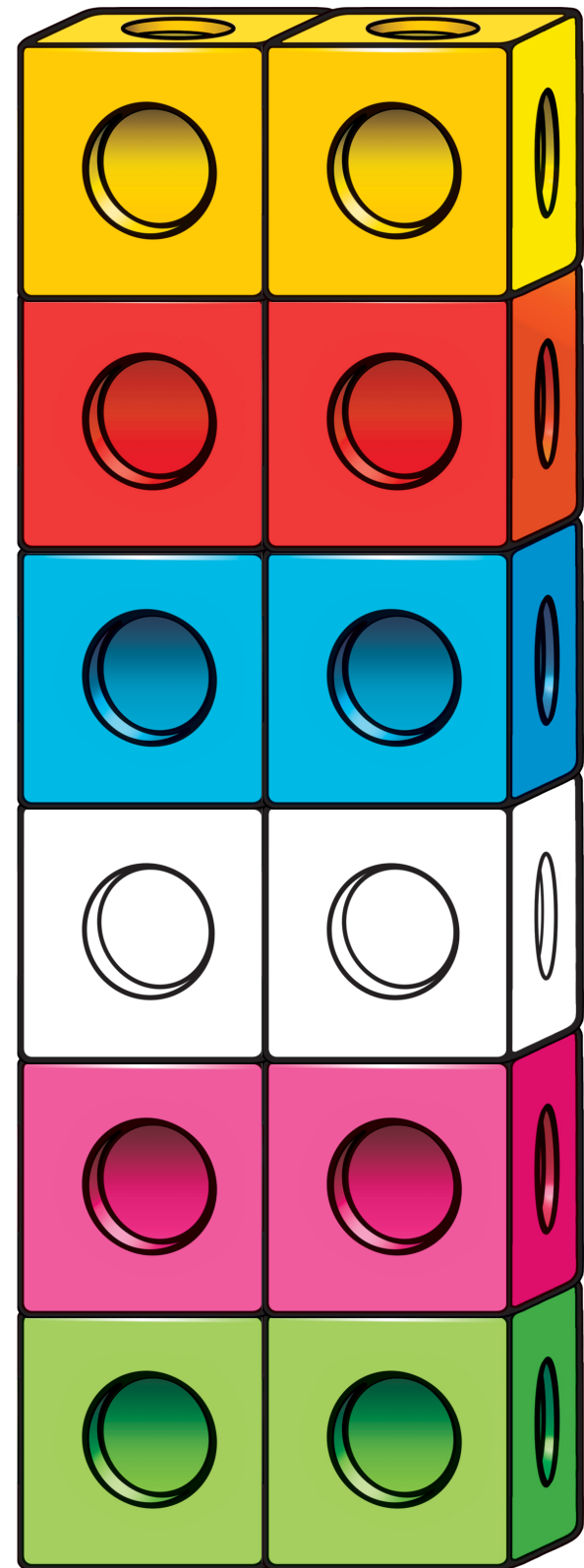
12



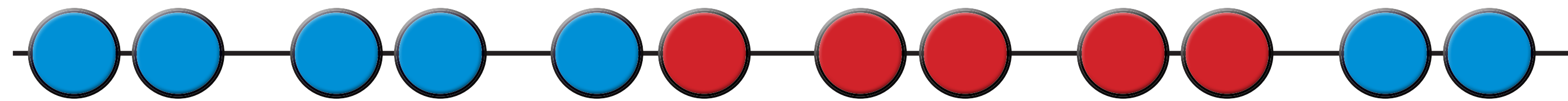
$$12 \div 2 = 6$$



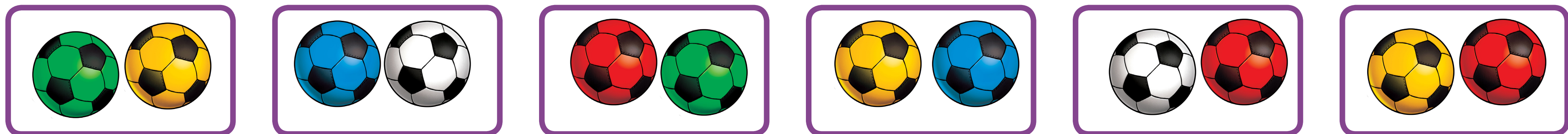
Grouping Model



“12 divided by 2”
“How many 2s in 12?”

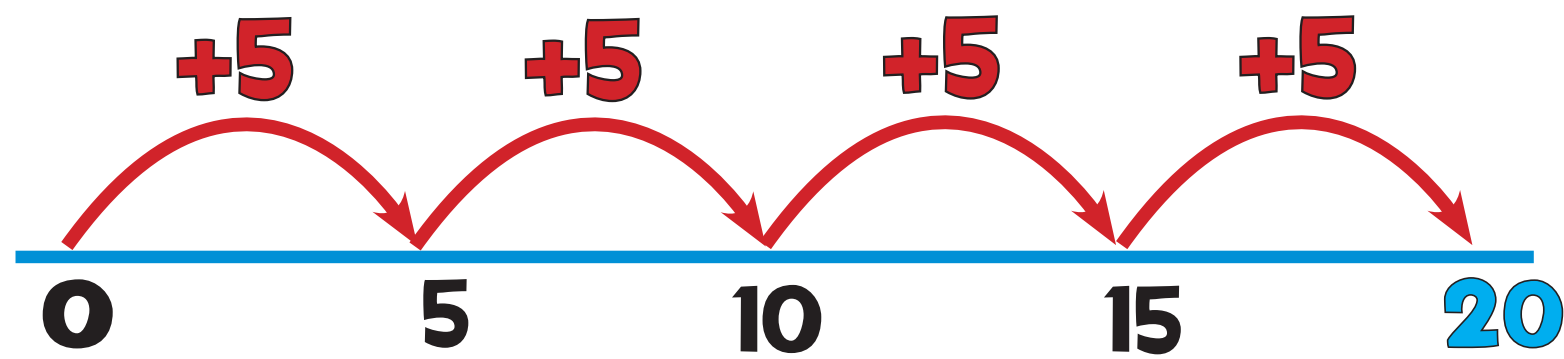
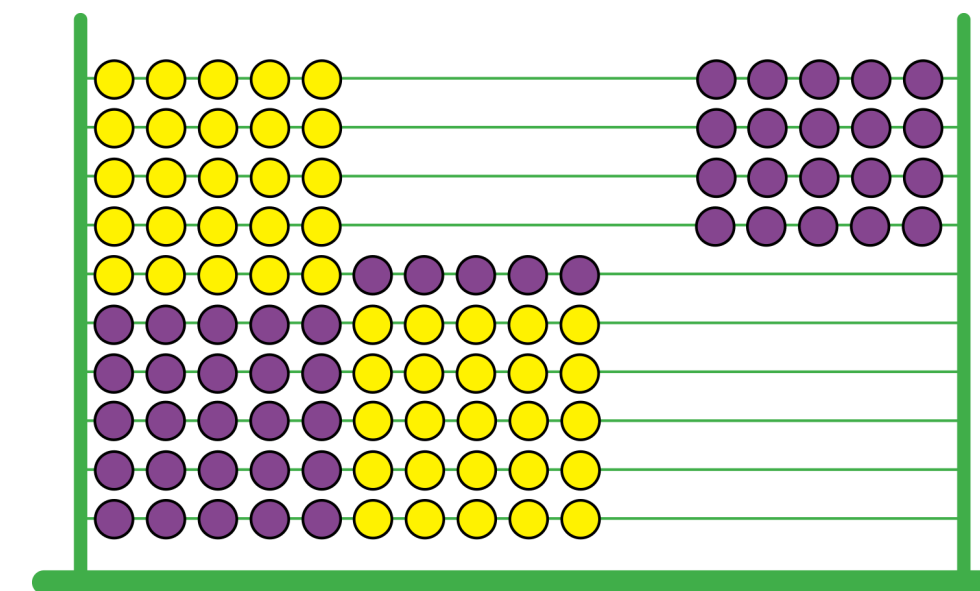
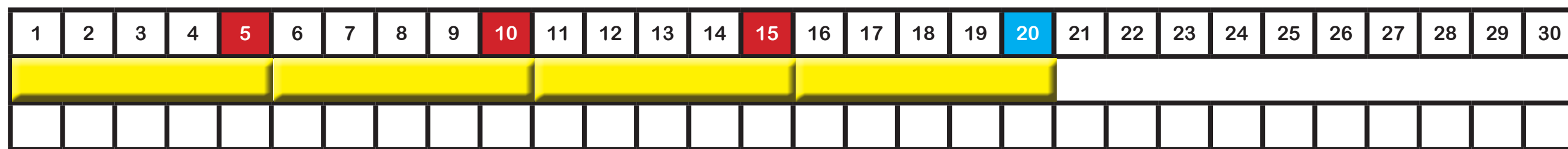
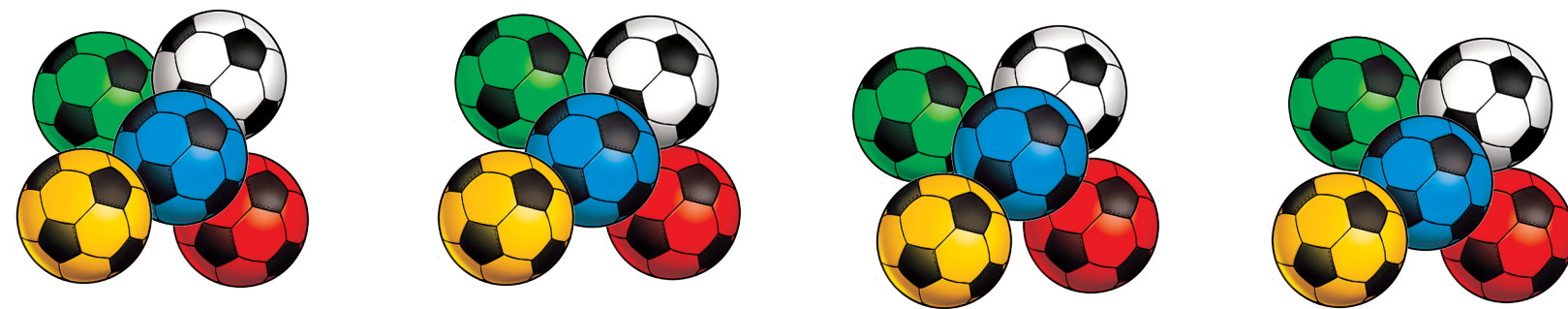
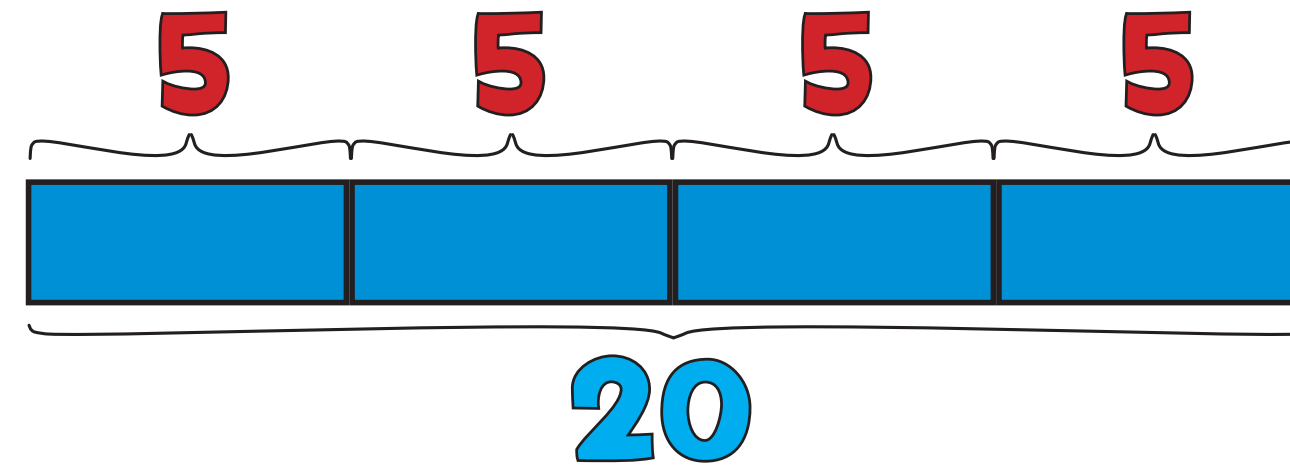
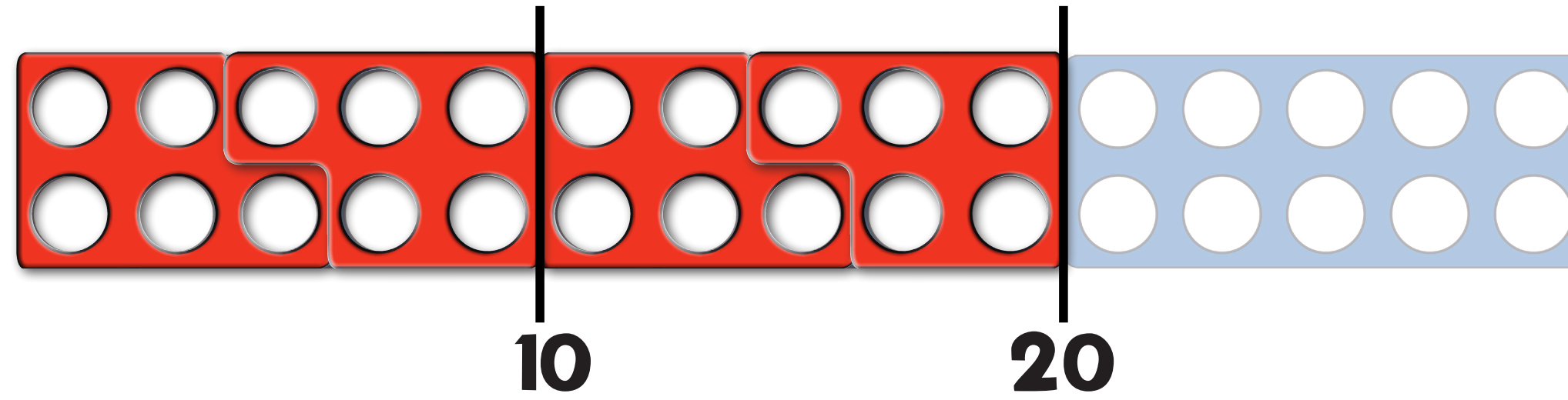
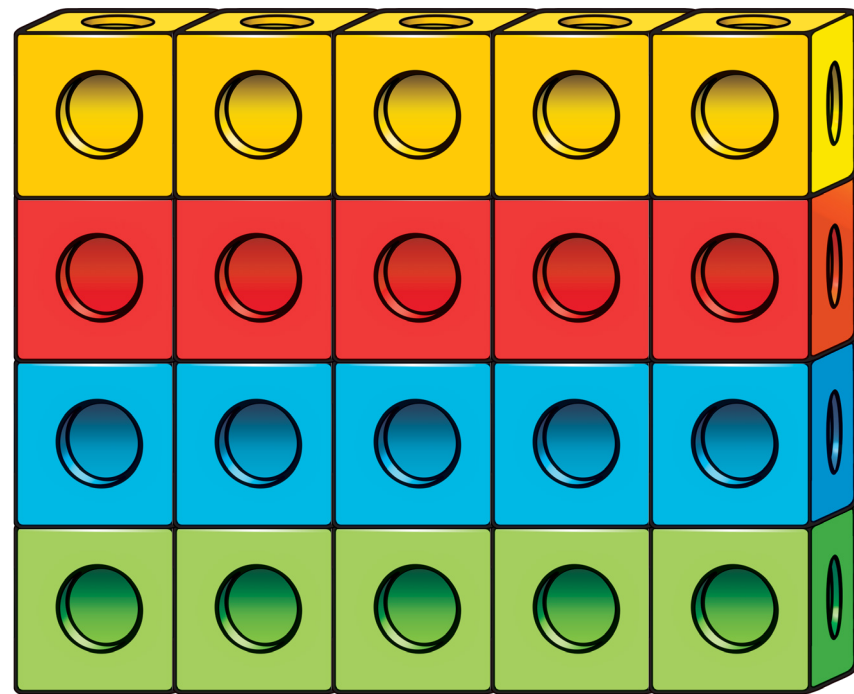


12 footballs in 6 groups of 2!



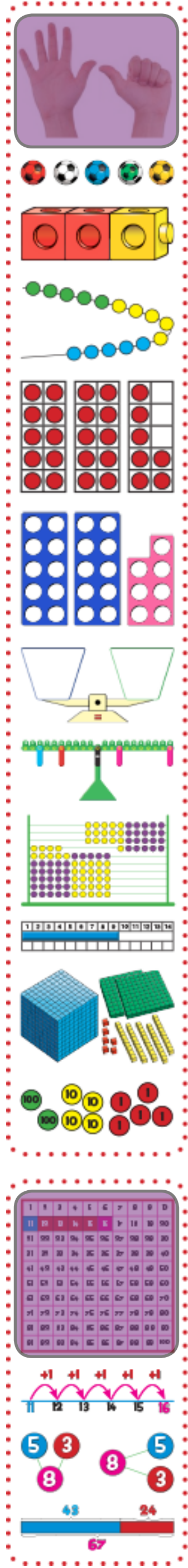
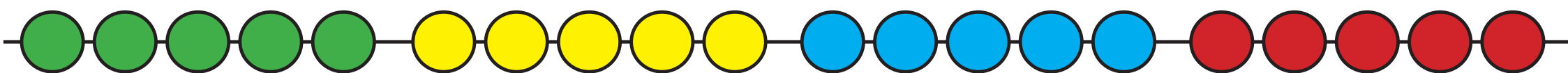
CPA
 ← reasoning →

$$20 \div 5 = 4$$



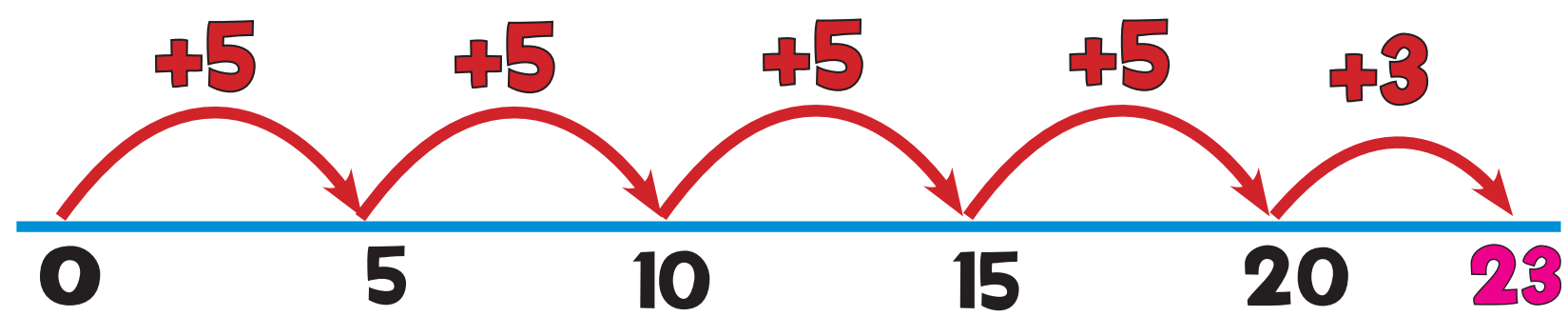
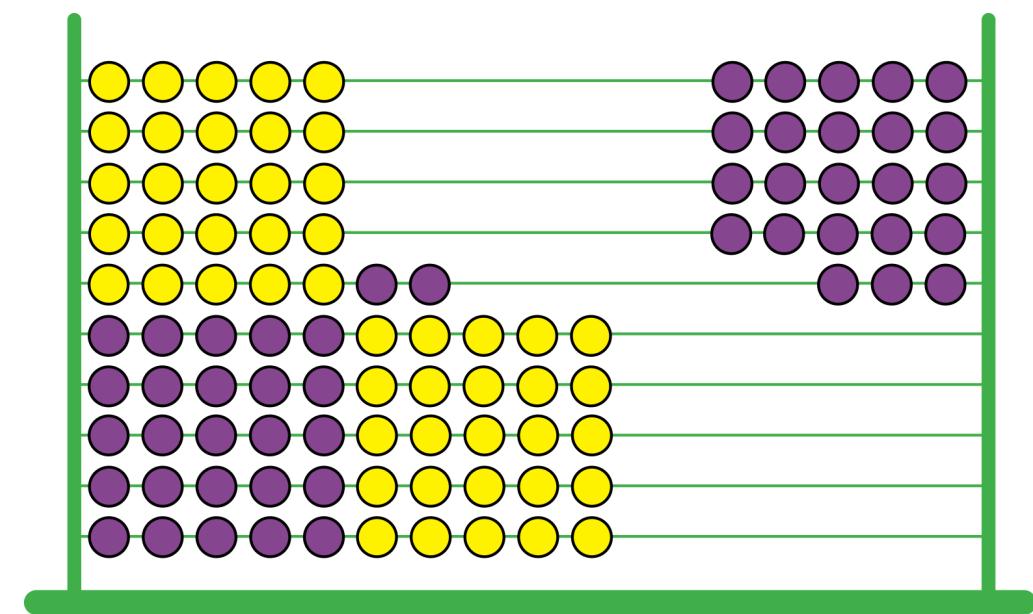
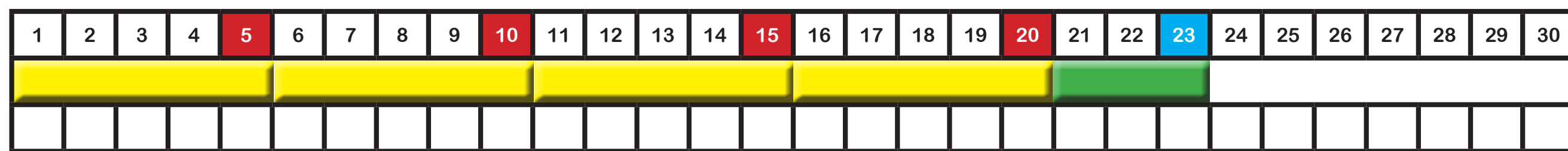
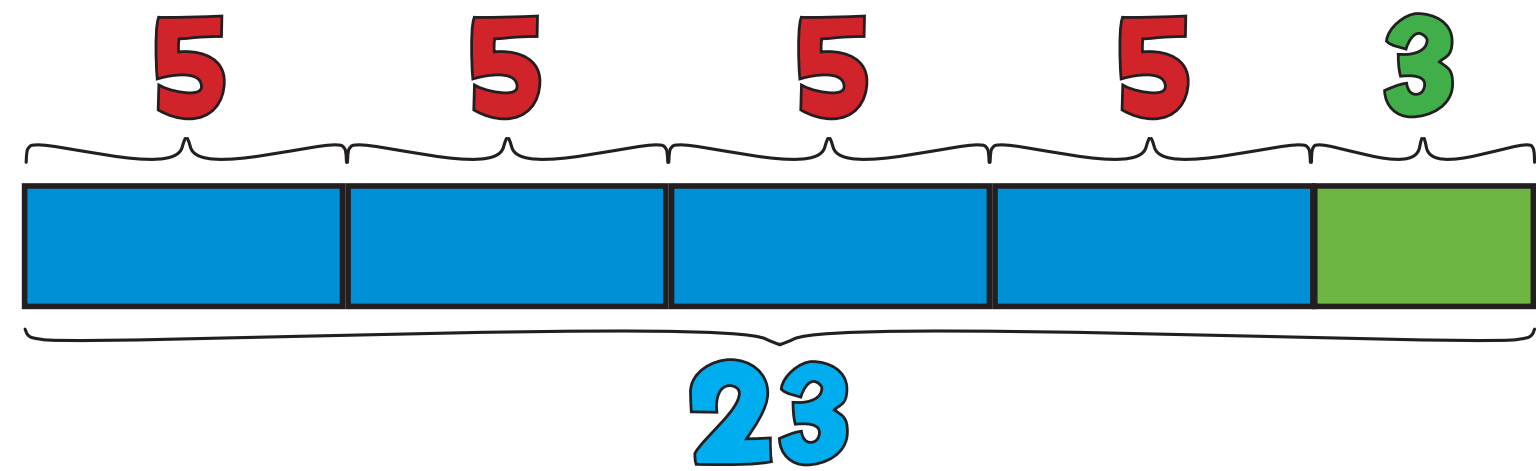
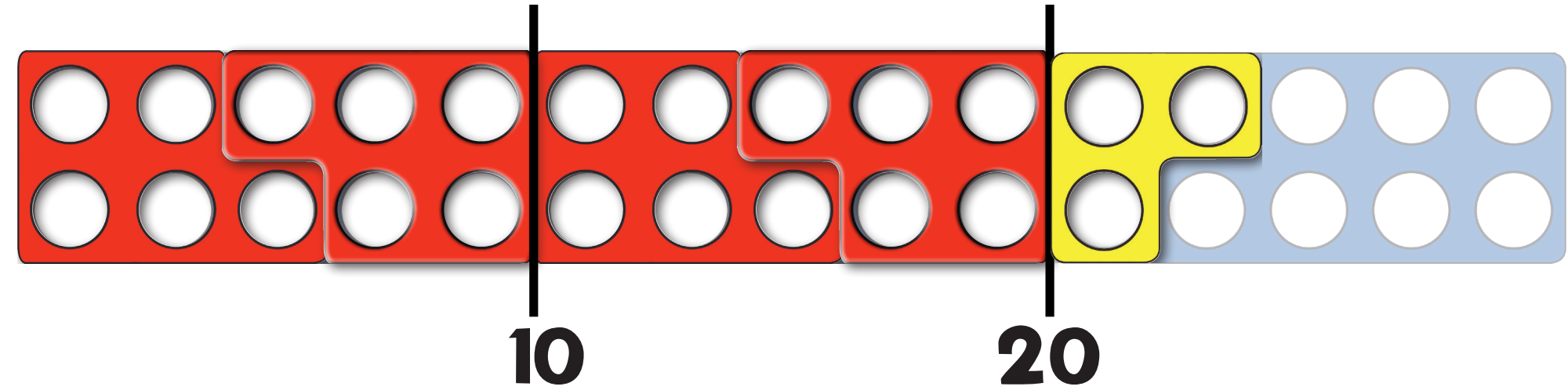
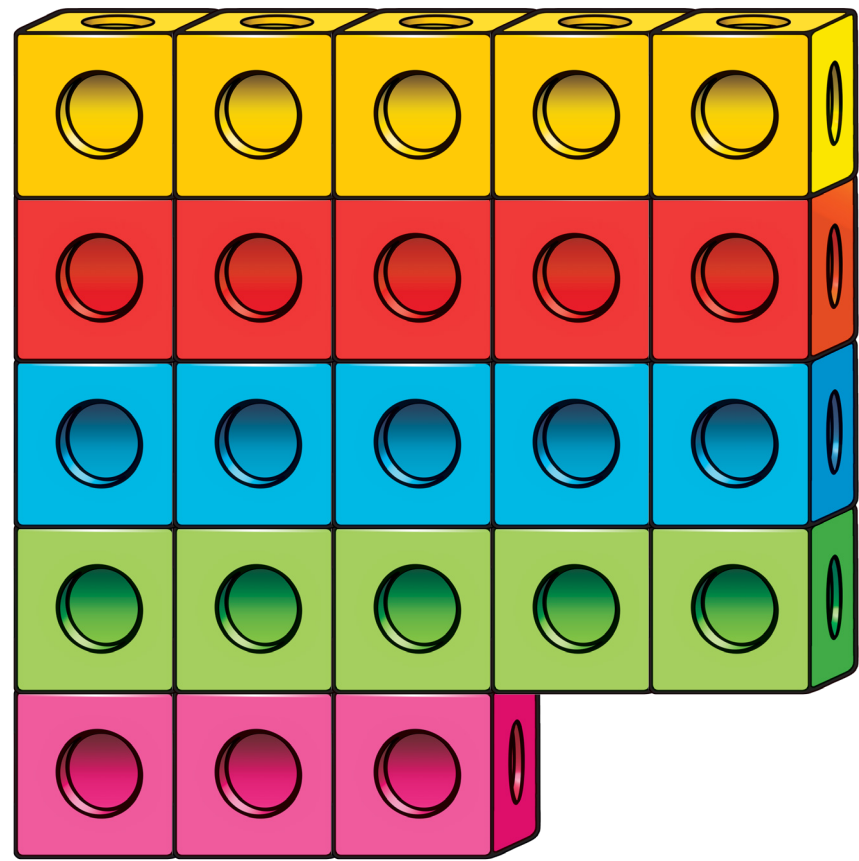
“How many 5s in 20?”

“20 divided by 5”



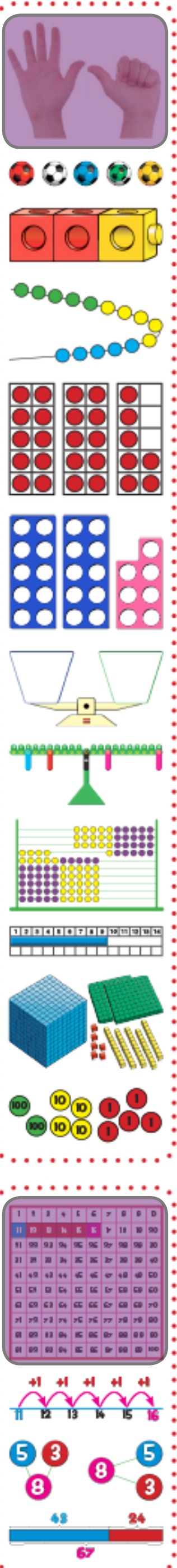
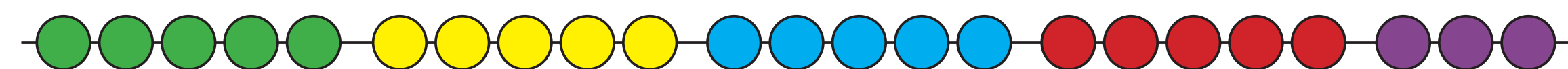
CPA
 ← reasoning →

$$23 \div 5 = 4r3$$

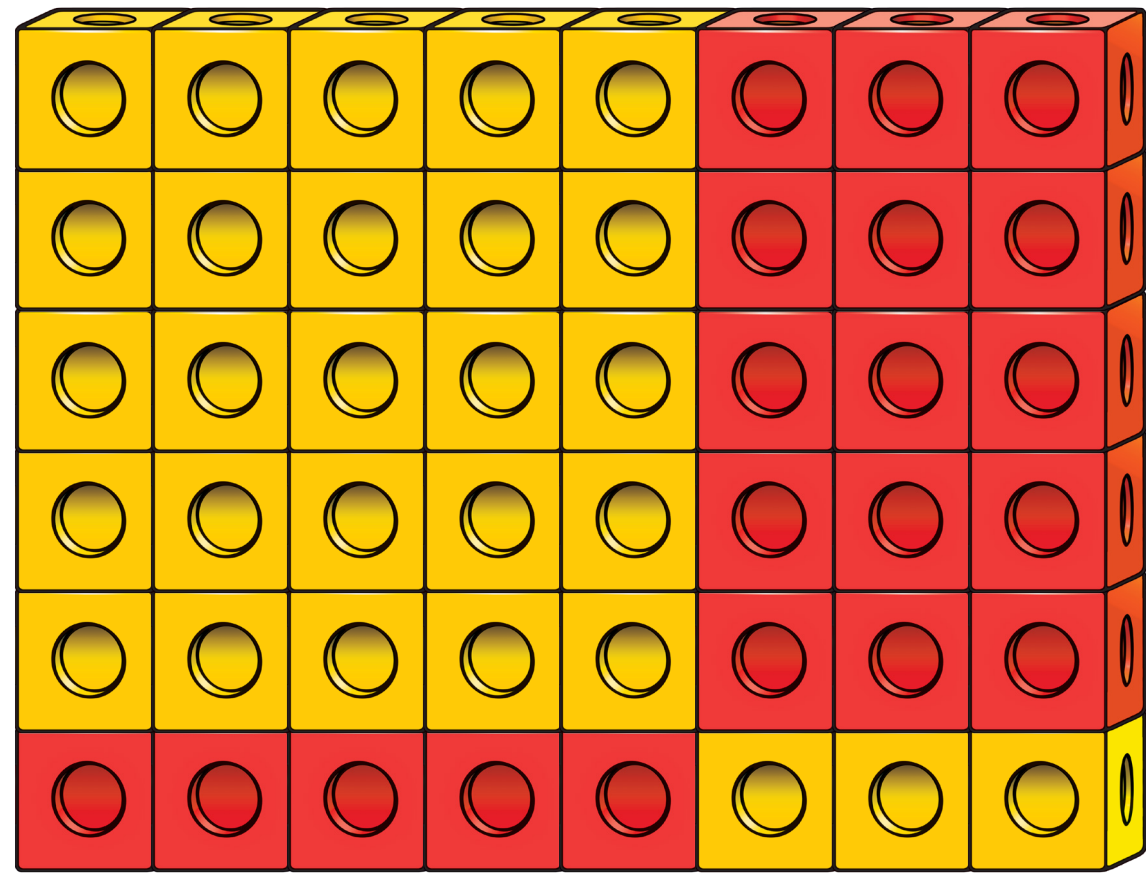


“How many 5s in 23?”

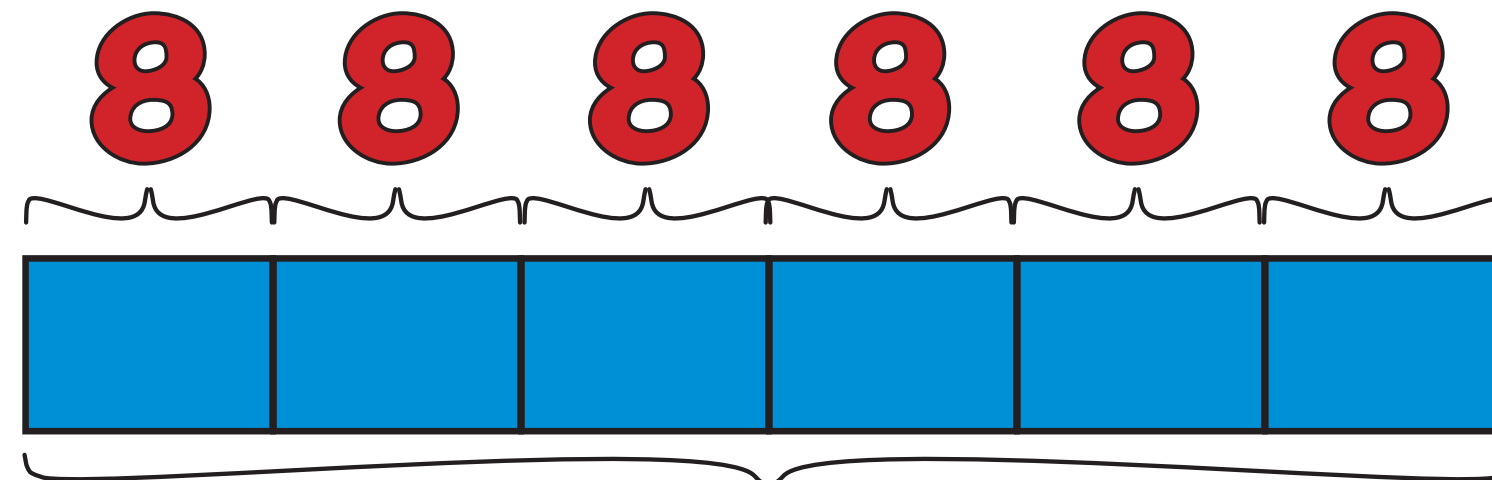
“23 divided by 5”



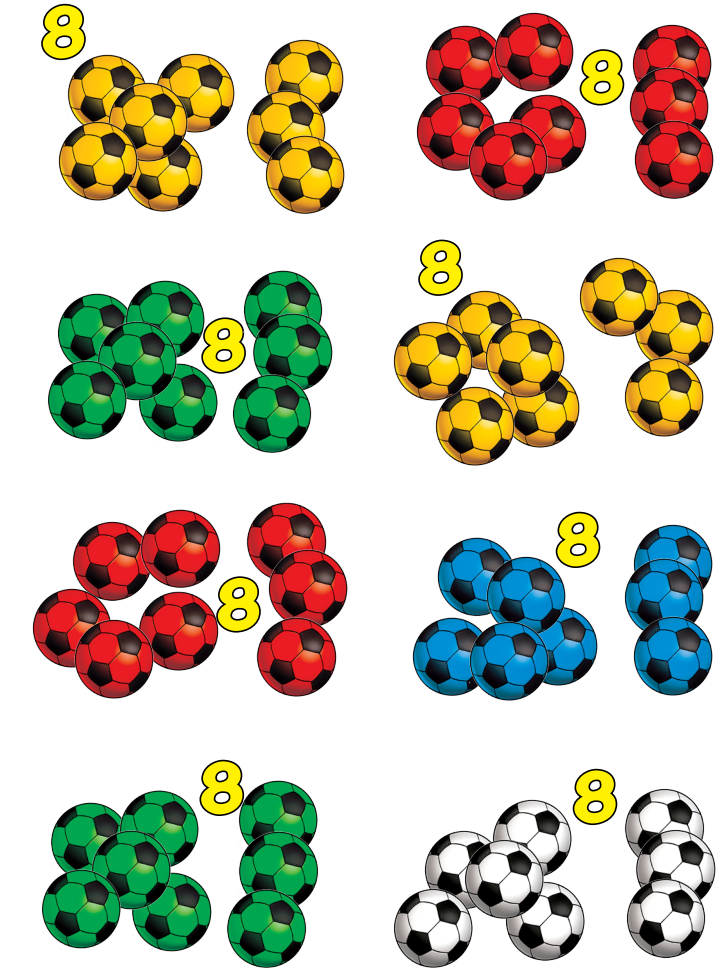
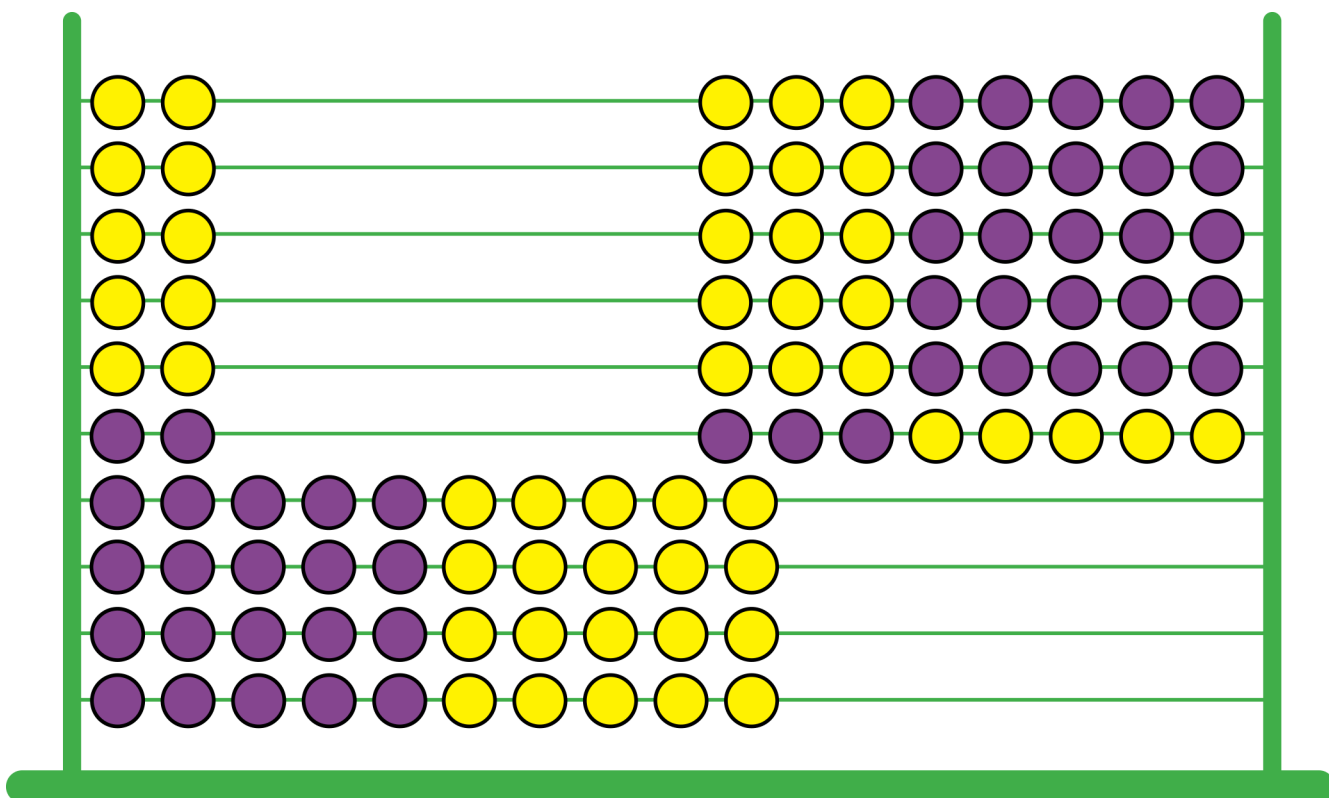
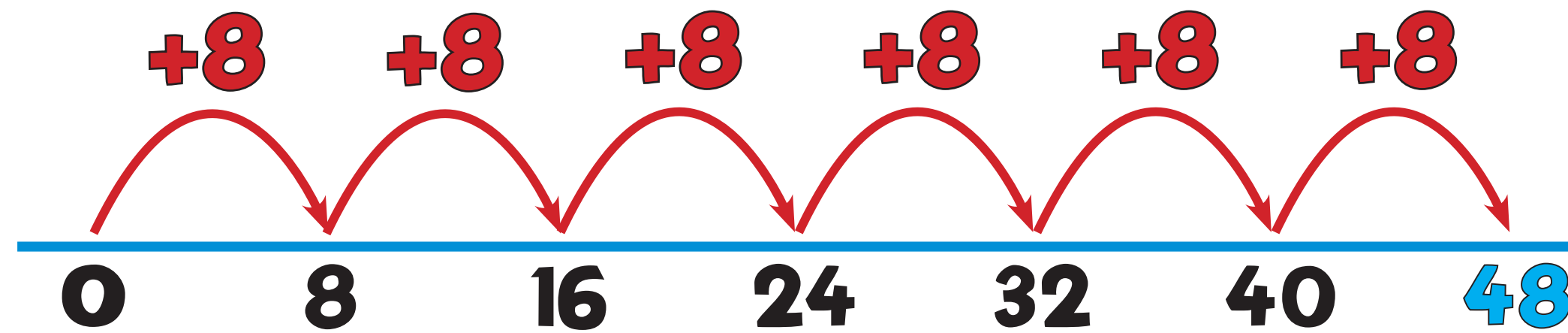
$$48 \div 8 = 6$$



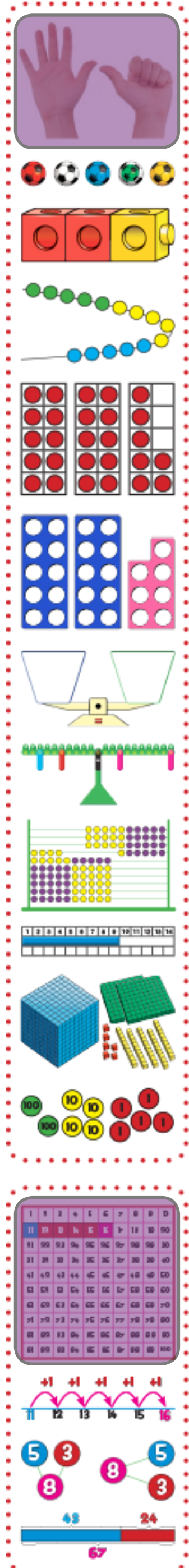
“How many 8s in 48?”
“48 divided by 8”



48



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
---	---	---	---	---	---	---	---	---	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----	----



$$72 \div 4 = 18$$

10 **8**

4 **40** **32**

$$\begin{array}{r} 40 \\ \downarrow \\ 10 \end{array} + \begin{array}{r} 32 \\ \downarrow \\ 8 \end{array} = 18$$

Grouping Image!

1

Tens Ones

$4 \overline{) 72}$

3

Tens Ones

$4 \overline{) 72}^3$

Sharing Image!

2

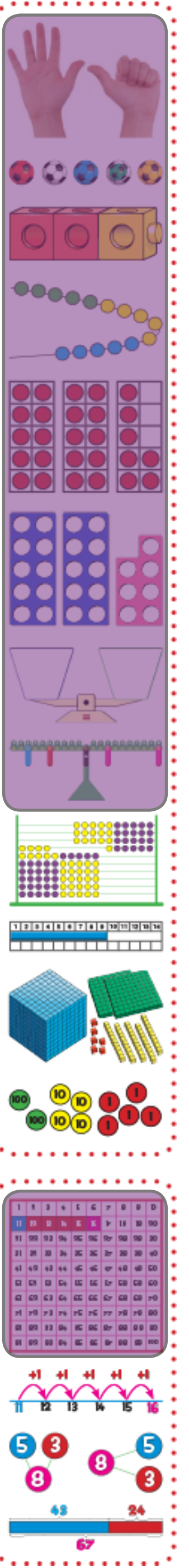
Tens Ones

$4 \overline{) 72}^1$

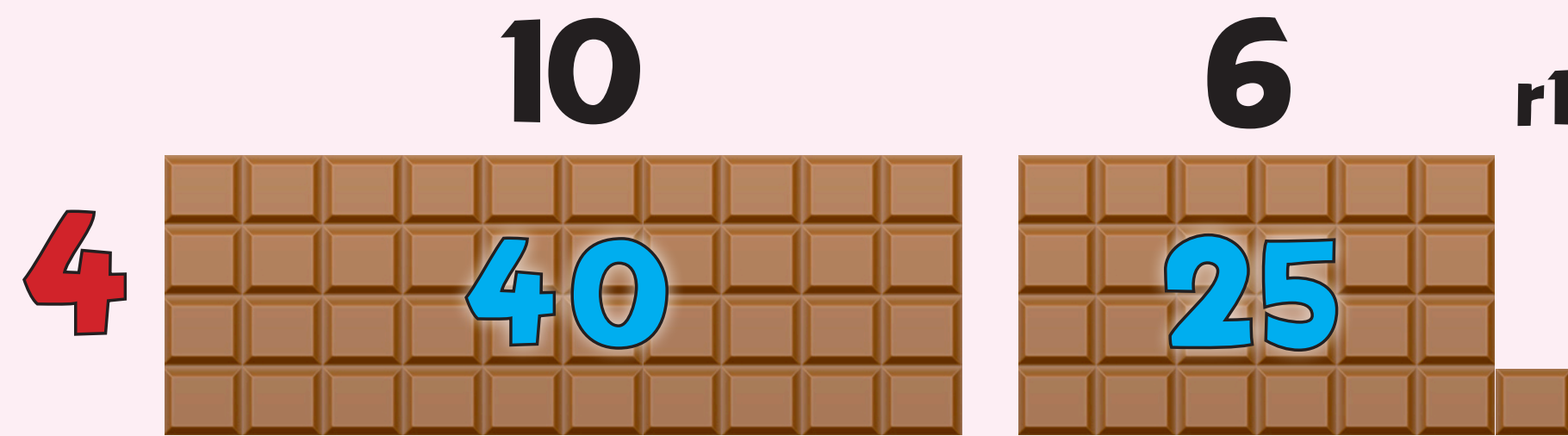
4

Tens Ones

$4 \overline{) 72}^{18}$



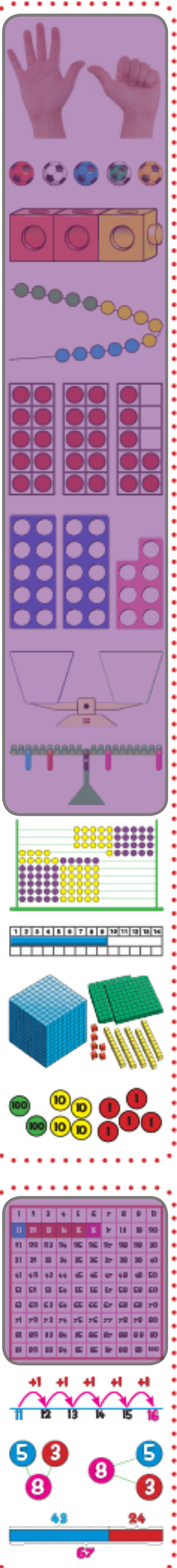
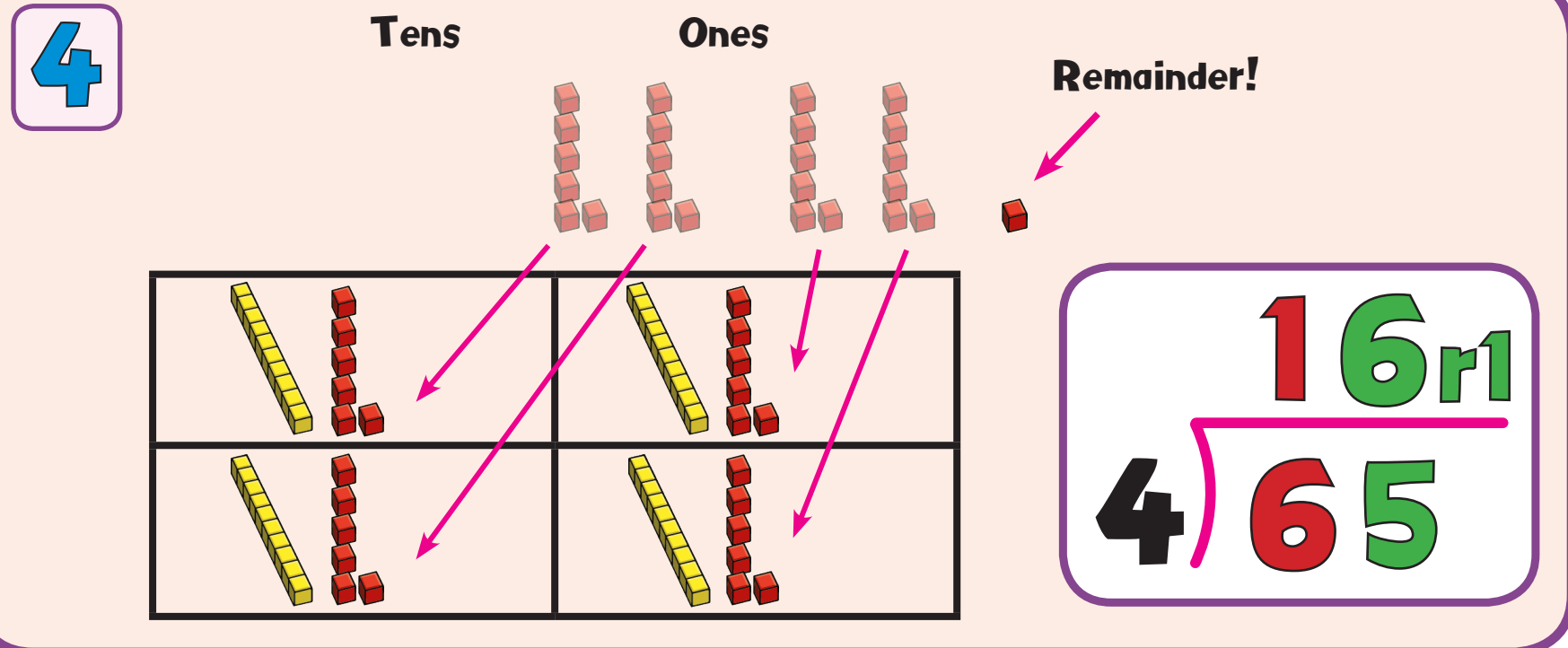
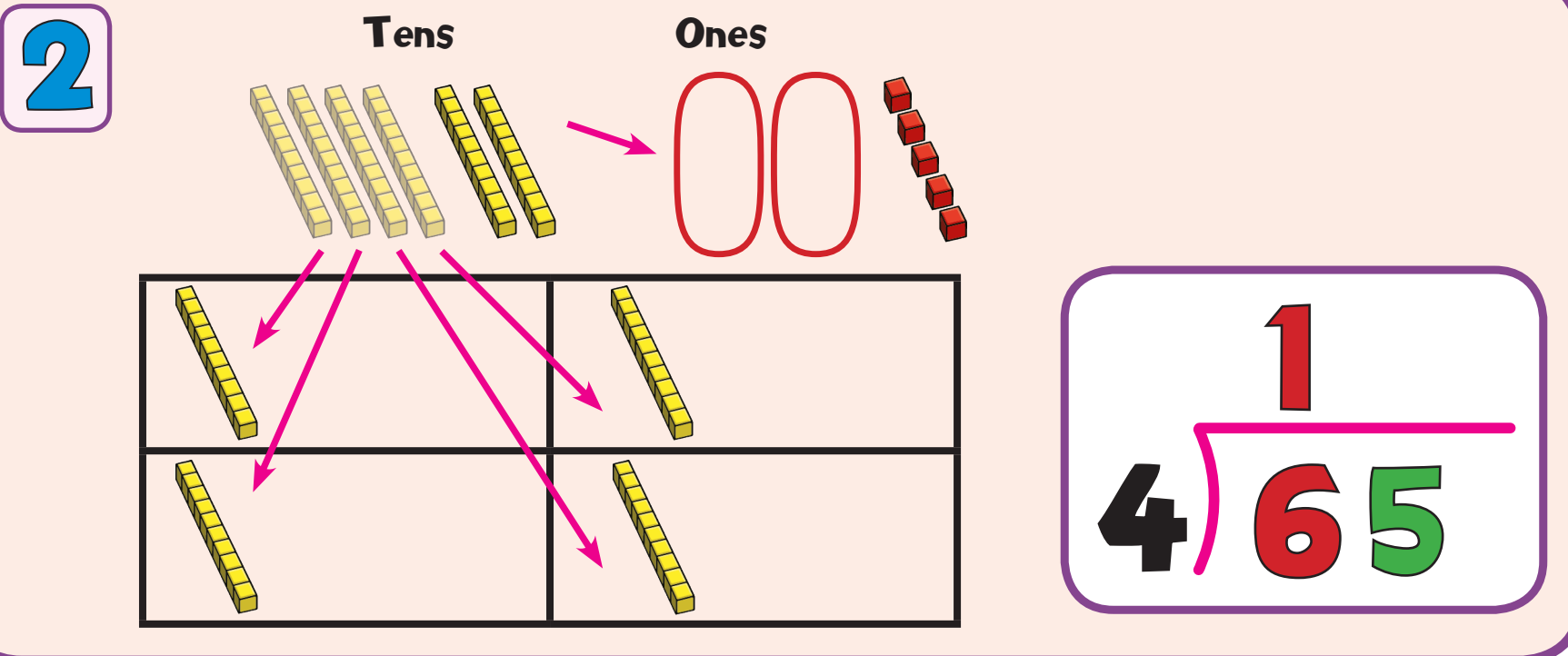
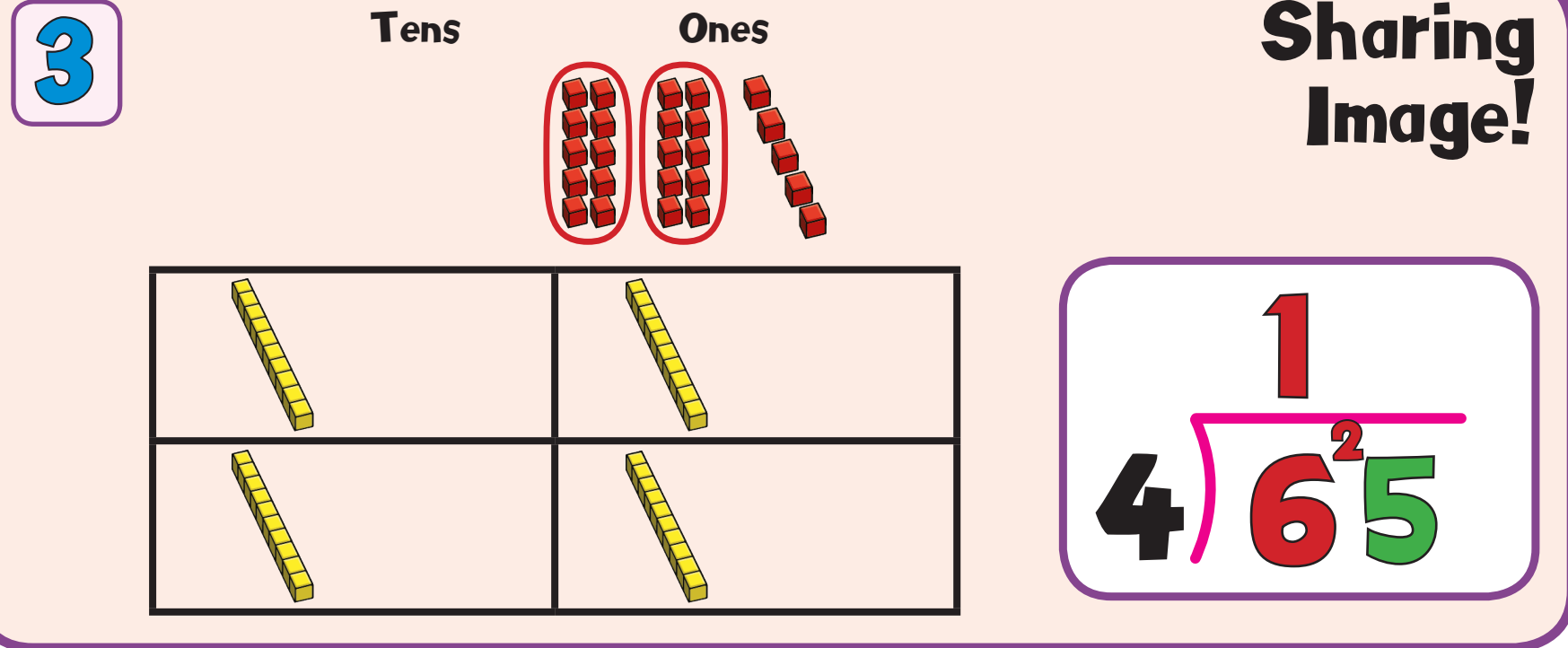
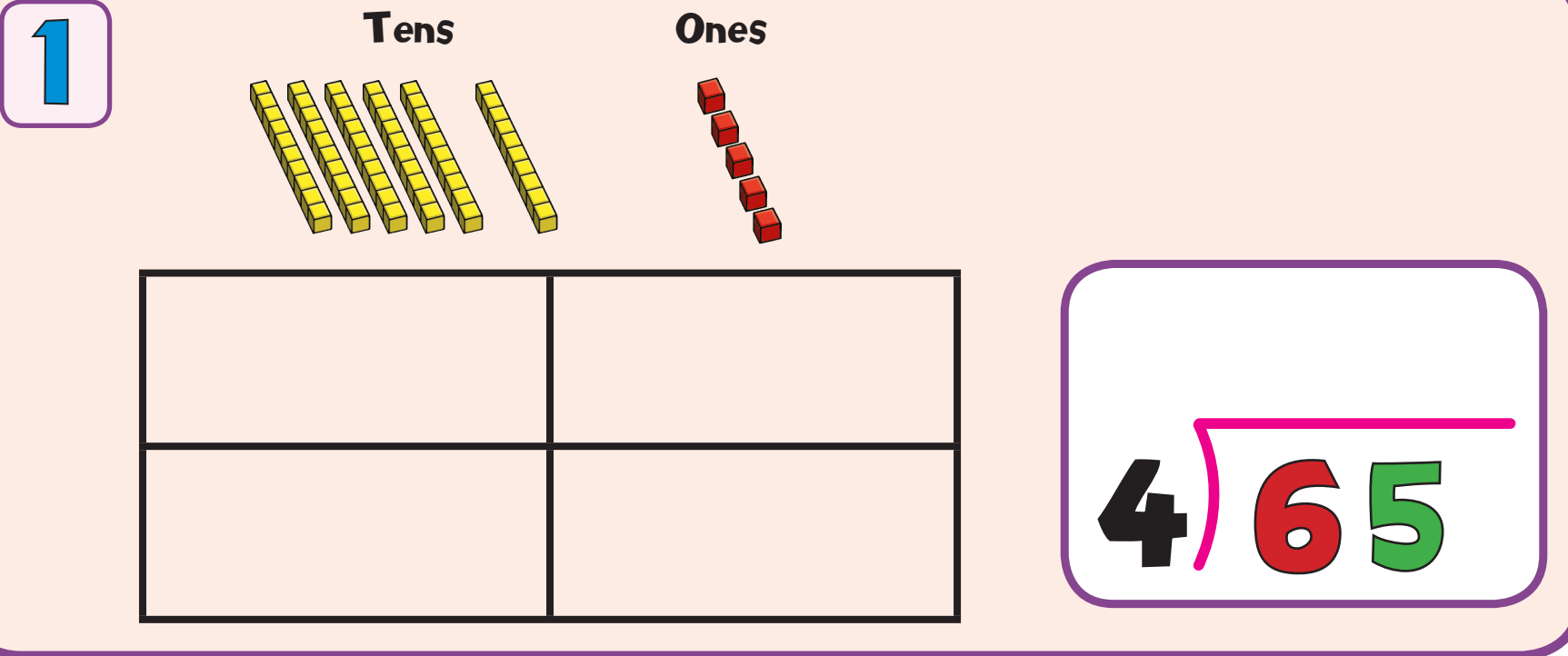
$$65 \div 4 = 16r1$$



40 + **25** **Grouping Image!**

↓ ↓ ÷ **4**

10 + **6r1** = **16r1**



$$136 \div 4 = 34$$

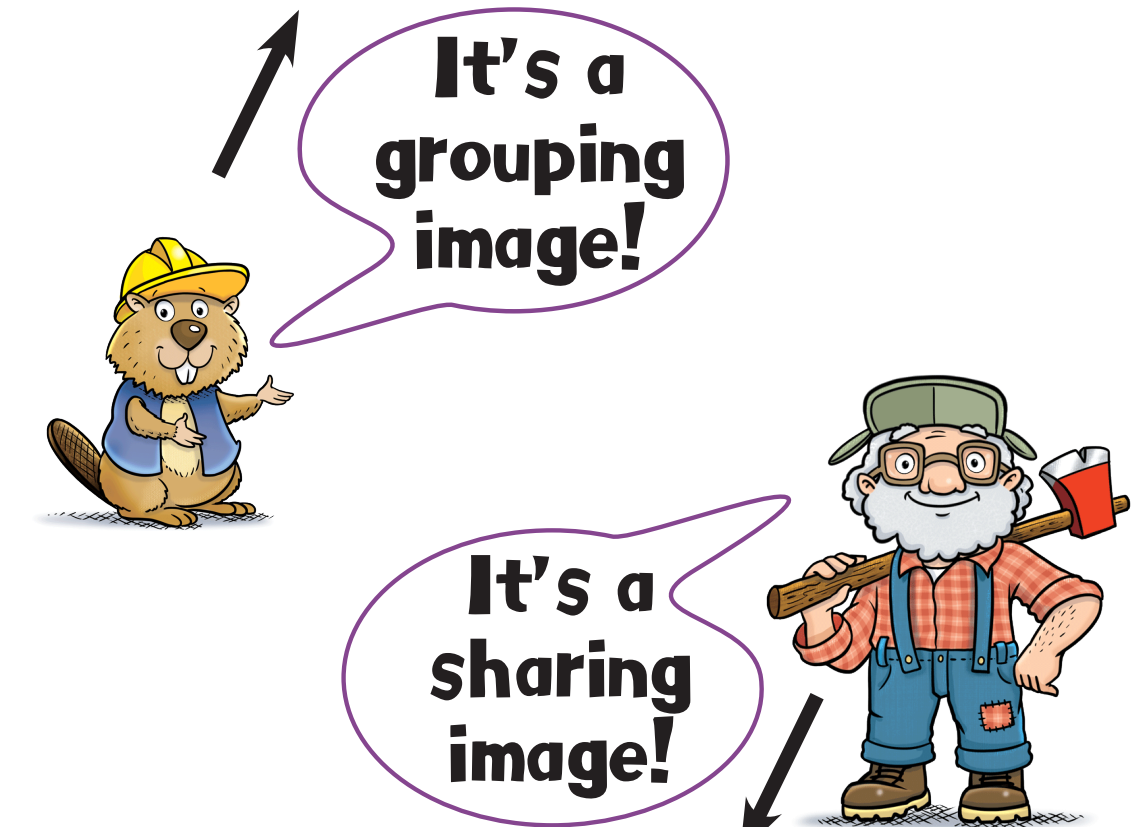


$$120 + 16 \div 4 = 30 + 4 = 34$$

1 Hundreds Tens Ones

$$4 \overline{)136}$$

3 Tens Ones

$$4 \overline{)136}$$


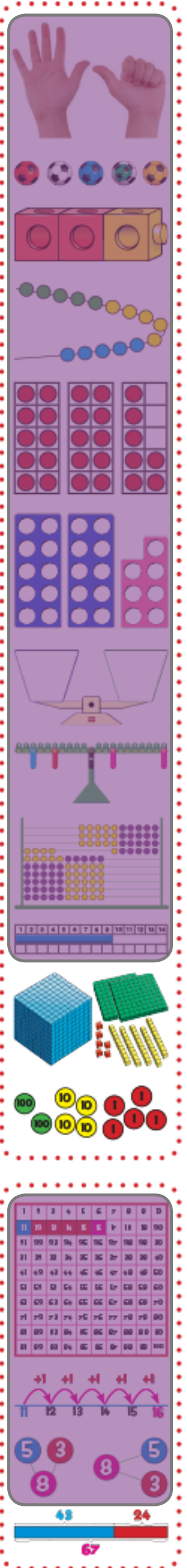
2 Hundreds Tens Ones

$$4 \overline{)136}$$

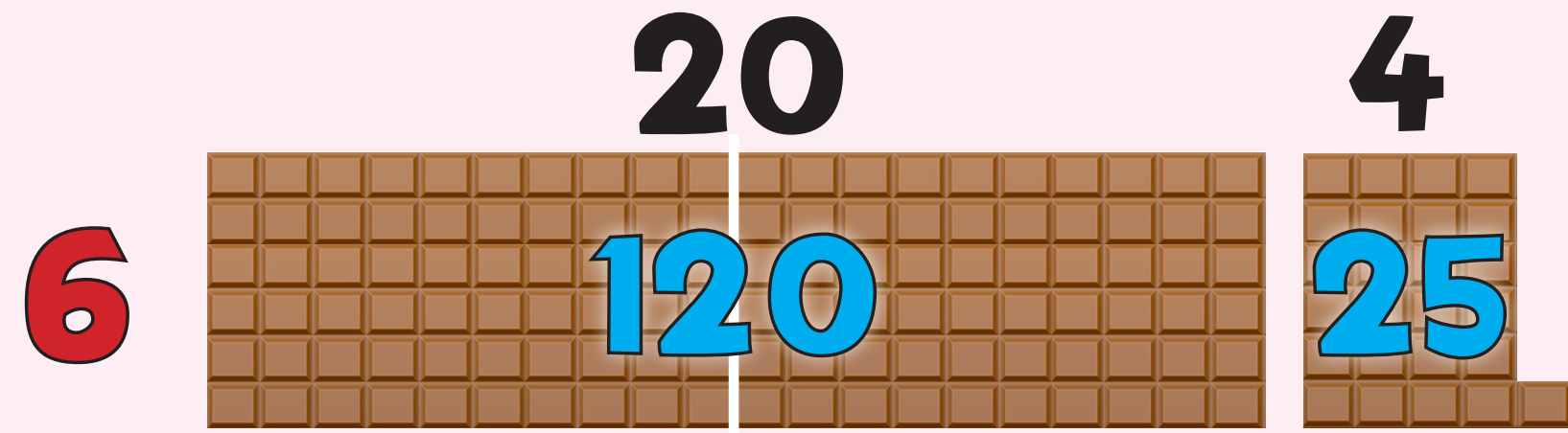
4 Tens Ones

$$4 \overline{)136}$$

5 Tens Ones

$$4 \overline{)136}$$


$$145 \div 6 = 24 r1$$



$$120 + 25 \div 6 = 20 + 4r1 = 24r1$$

1 Hundreds Tens Ones

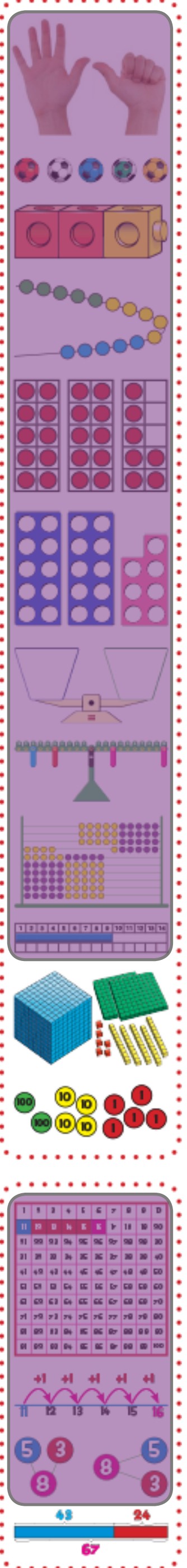
3 Tens Ones

It's a grouping image!

2 Hundreds Tens Ones

4 Tens Ones

5 Ones Remainder!



$$536 \div 4 = 134$$

1

Hundreds Tens Ones

$$4 \overline{)536}$$

2

Hundreds Tens Ones

$$4 \overline{)1536}$$

3

Hundreds Tens Ones

$$4 \overline{)136}$$

4

Tens Ones

Sharing Image!

$$4 \overline{)136}$$

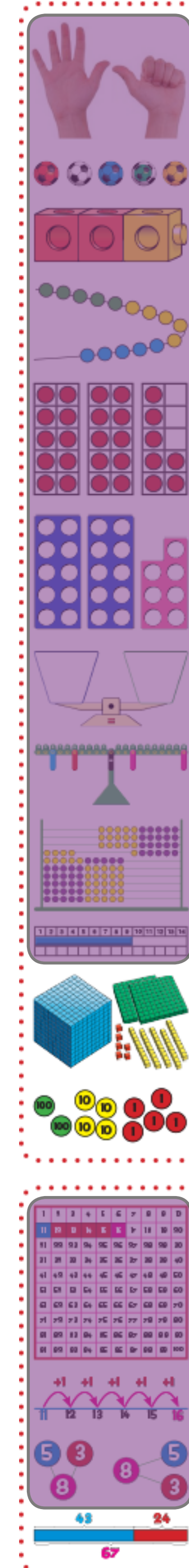
5

Ones

$$4 \overline{)136}$$

6

Ones

$$4 \overline{)136}$$


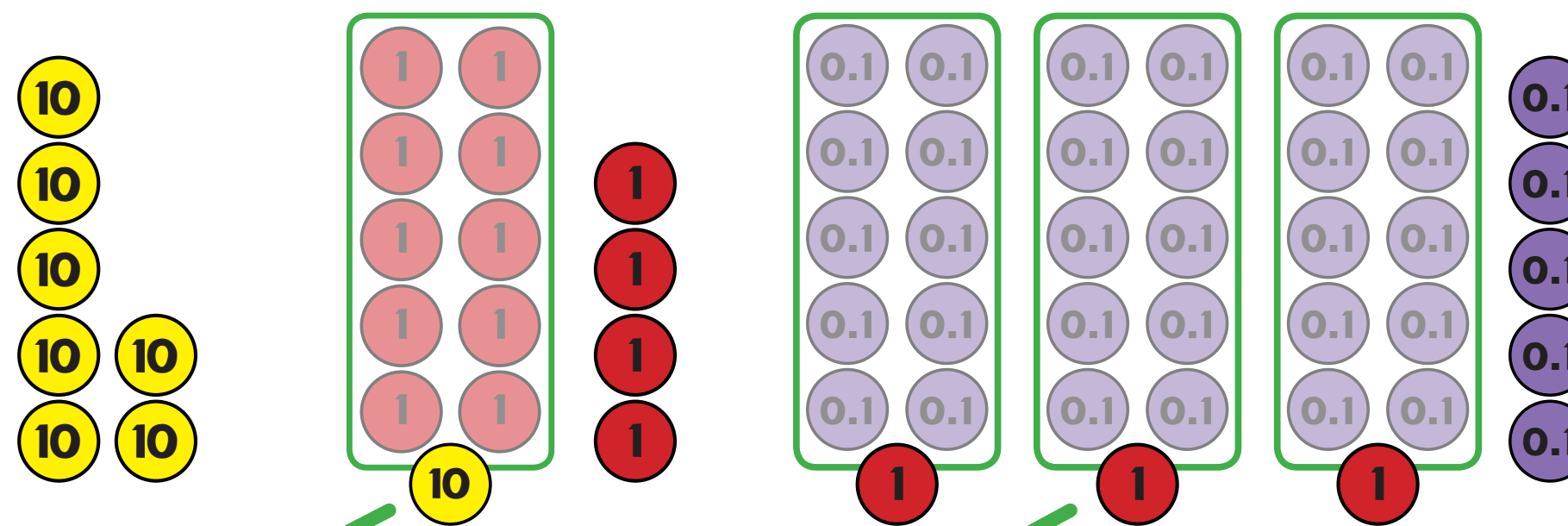
CPA
reasoning

$$87.5 \div 7 = 12.5$$

$$70 + 14 + 3.5 = 87.5$$

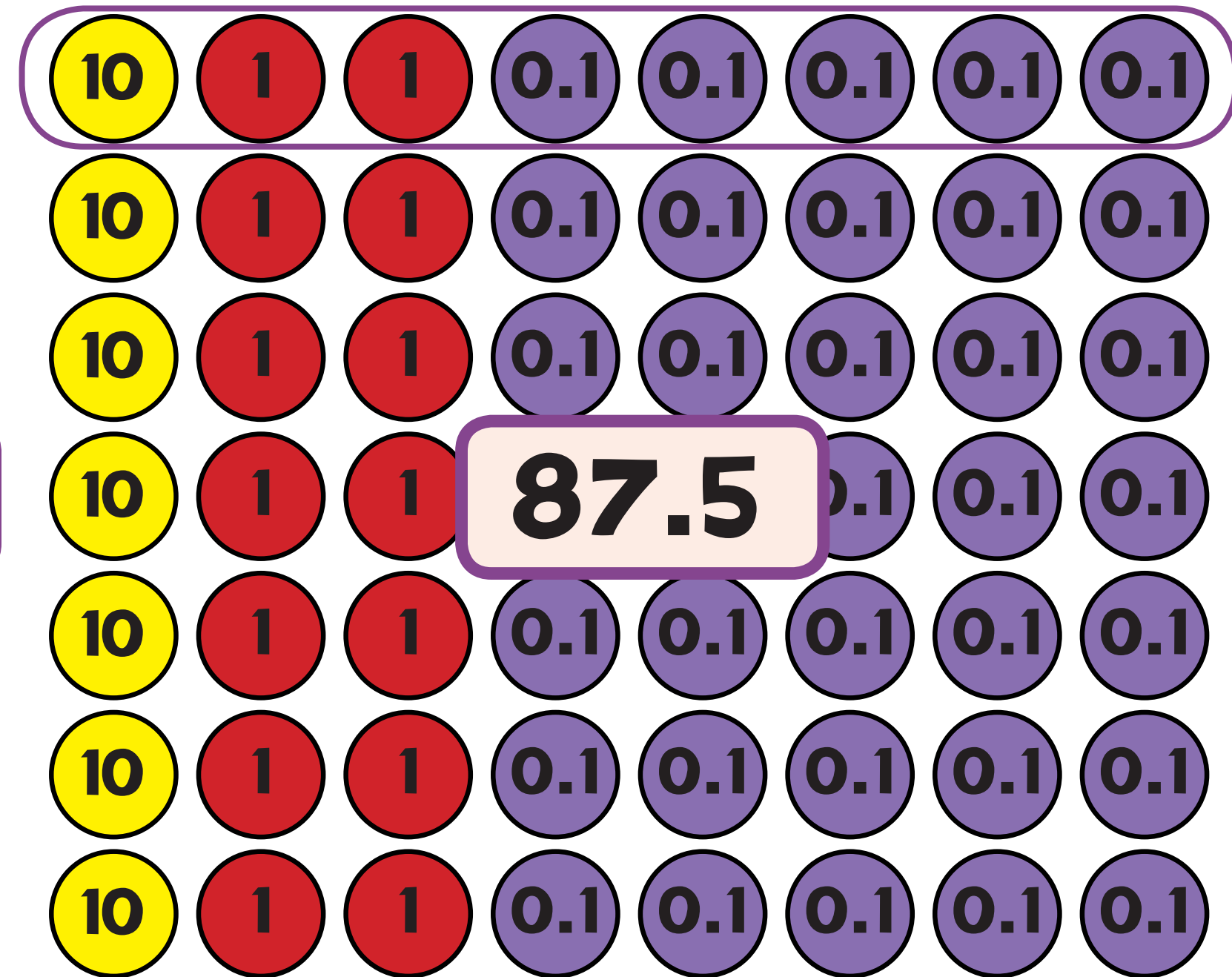
$$\downarrow \quad \downarrow \quad \downarrow$$

$$10 + 2 + 0.5 = 12.5$$



10 2 0.5

7



$$\begin{array}{r} 12.5 \\ 7 \overline{) 87.5} \end{array}$$

