



Sense of Number



Expanded Visual Calculation Policy

Concrete & Pictorial Policy



Hawkesley Church Primary Academy July 2019



Graphic Design by Dave Godfrey
Compiled by the Sense of Number Maths Team

For sole use within Hawkesley Church Primary Academy.

'A picture is worth 1000 words!' www.senseofnumber.co.uk

CPA
↔
reasoning





Guide to using the



Concrete and Pictorial Visual Calculation Policy

CPA

↔
reasoning

C: Concrete

(Real objects that children can touch and manipulate.)

P: Pictorial

(Images that help children picture the mathematics.)

A: Abstract

(Calculations written using symbols.

e.g. $2 + 2 = 4$.)

A3 Wallchart Listing 84 Posters:

- 2:** Introductory Pages
- 5:** Resources Summary
- 7:** Operation Overviews
- 12:** Addition Posters
- 32:** Subtraction Posters
- 59:** Multiplication Posters
- 70:** Division Posters



The Full Sense of Number Visual Calculation Policy Package provides a comprehensive visual representation of a school's Calculation Policy.

- 1: CPVCP** Concrete and Pictorial VCP - The foundation of the policy, featuring key models and images to help children gain deep understanding of the abstract procedures.
- 2: WSVCP** Written Strategies progression from jottings to formal written methods from Y1 to Y6.
- 3: MSVCP** Mental Strategies progression across KS1 and KS2.
- 4: ECPD** Editable Calculation Policy Document - a comprehensive written explanation of a school's calculation policy, featuring thumbnails of the posters from the three documents above.

Typical uses:

- Classroom:** The posters are printed out (e.g. A4) and the appropriate slides are displayed for continual reference or on a working wall. Posters are used on the interactive whiteboard.
- Reference:** The summary overviews are printed out and inserted in the teacher's planning folder.
- Parents:** The posters are used to communicate to parents the methods being used within school.
- Website:** Screen grabs of slides from the VCP are inserted on a schools' maths webpages.

(PLEASE NOTE: the VCP should not be placed on school website for copyright reasons.)

A secure PDF copy of the Editable Calculation Policy may be placed on the school website.



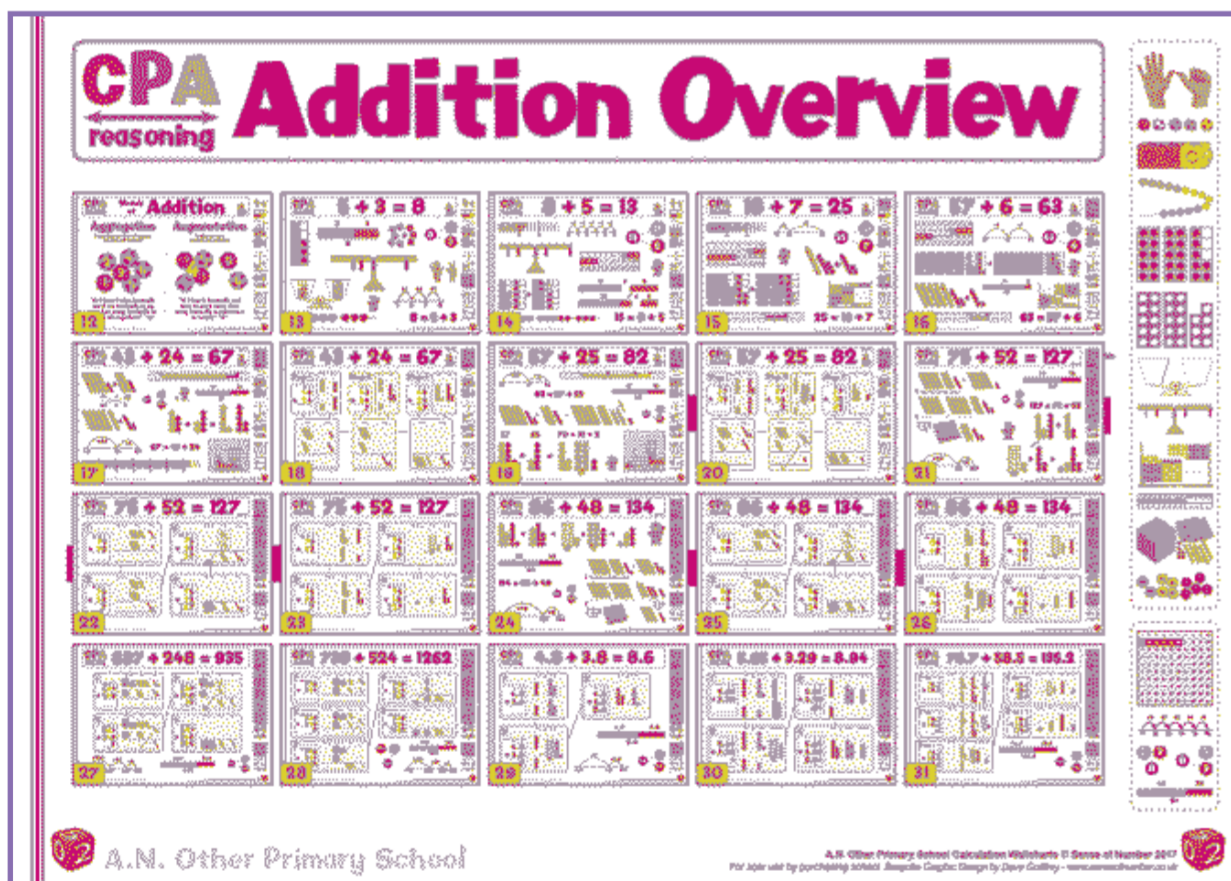
Expanded Visual Calculation Policy

The Expanded Visual Calculation Policy helps children and teaching staff achieve mastery of all aspects of calculation. It contains the following three documents:

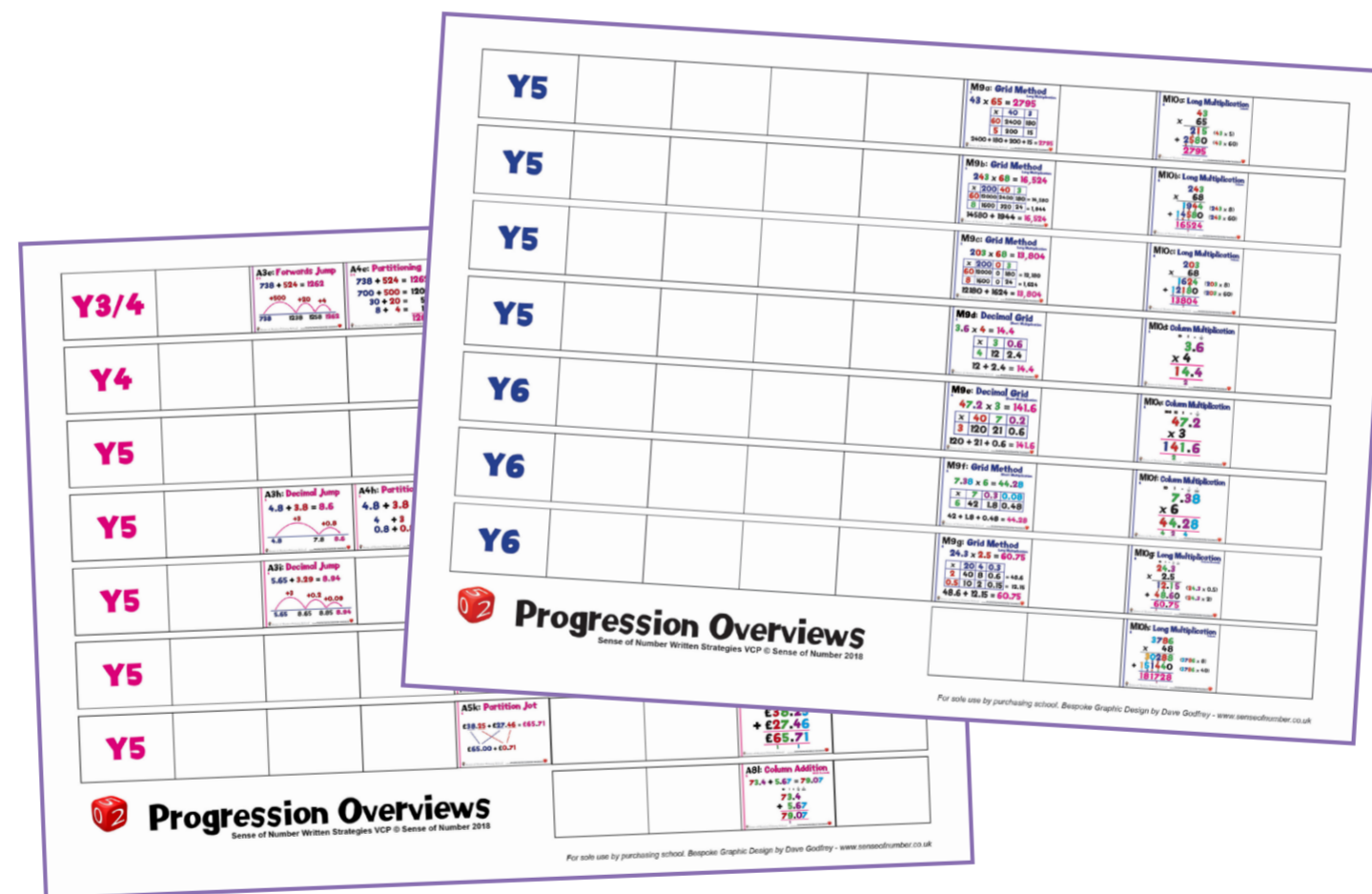
Concrete & Pictorial VCP

Written Strategies VCP

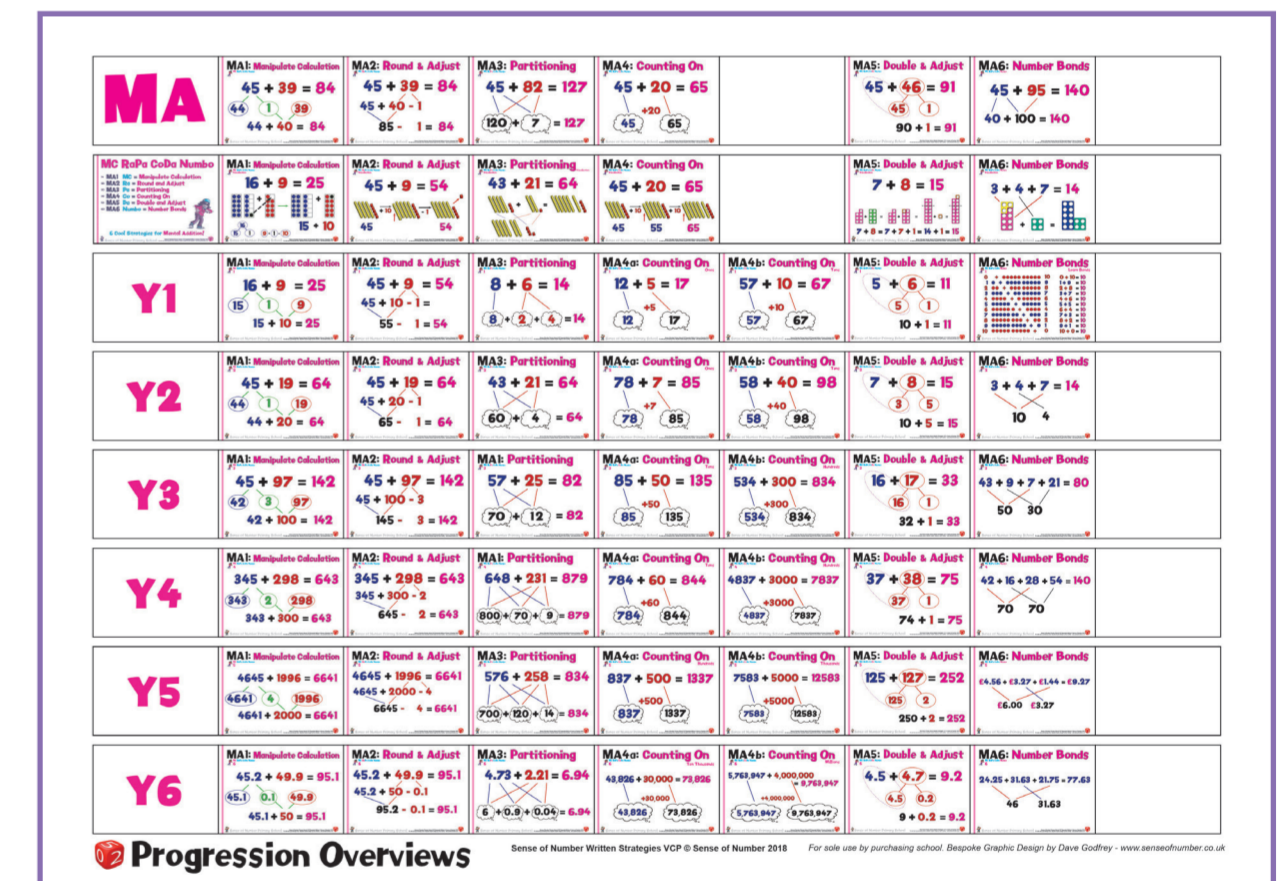
Mental Strategies VCP



84 A3 wallcharts showing the range of models and images that help children to understand and master calculation strategies.



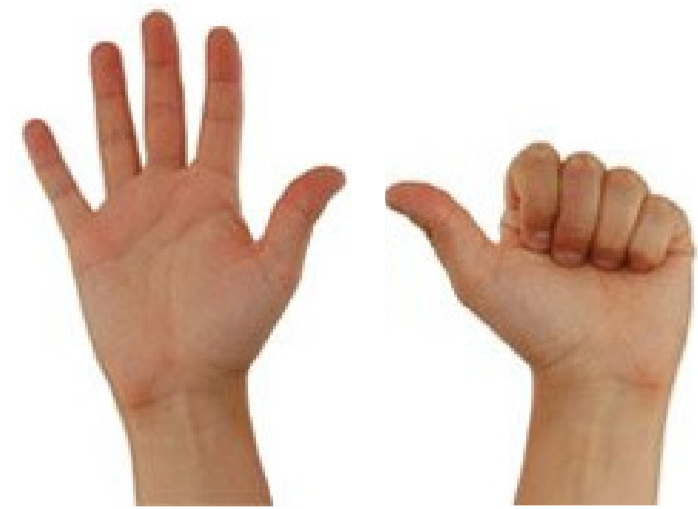
271 A4 posters showing the progression of written strategies (from Y1 to Y6) for all 4 operations in line with the National Curriculum.



214 A4 posters showing the progression of mental strategies (from Y1 to Y6) for all 4 operations in line with the National Curriculum.

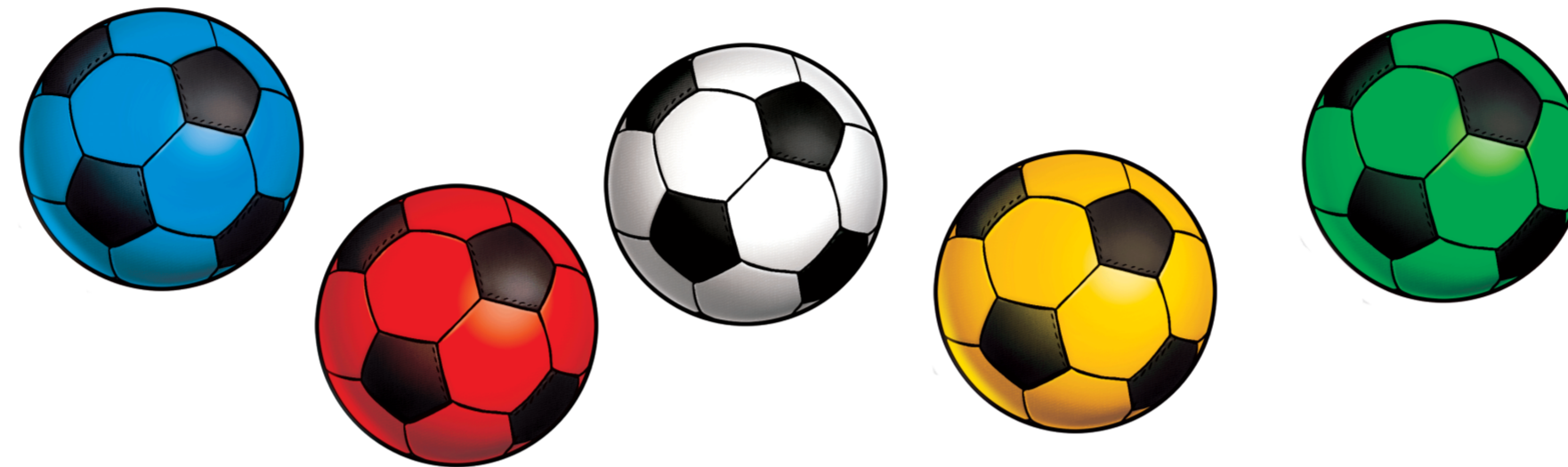


Code	Section	Concrete & Pictorial (84 A3 Wallcharts)		Written VCP (271 A4 Posters)		Mental VCP (215 A4 Posters)	
		Number of Wallcharts	Wallchart Numbers	No. of Posters	Poster Numbers	No. of Posters	Poster Numbers
	Policy Introduction Slides	4	1-4	4	1-4	4	1-4
	Introductory Posters	3	5-7	9	5-13		
	Operation Overviews	4	8-11	13	14-26	8	5-12
C	Counting Policy			15	27-41		
A	Addition	20	12-31	54	42-103		
MA	Mental Addition					55	13-67
S	Subtraction	27	32-58	48	104-169		
MS	Mental Subtraction					63	68-130
M	Multiplication	11	59-69	39	170-209		
MM	Mental Multiplication					46	131-176
D	Division	15	70-84	55	210-264		
MD	Mental Division					38	177-215
	Multiplication Tables			11	265-275		
	Alternative layouts (Column & Number Lines)			29	276-306		



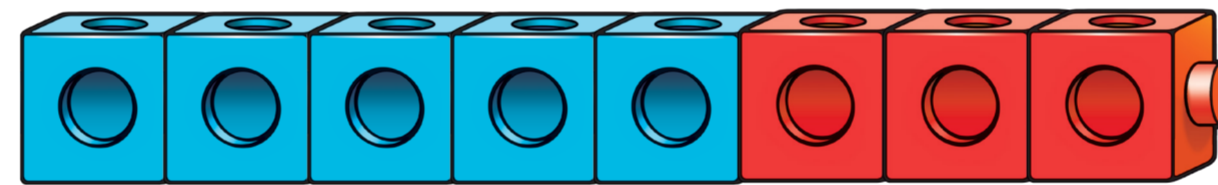
Fingers

Everyday Objects



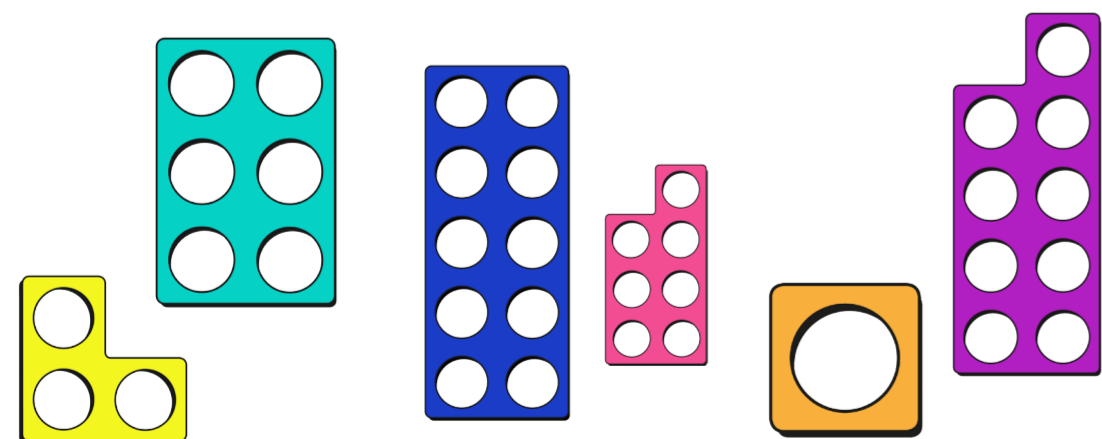
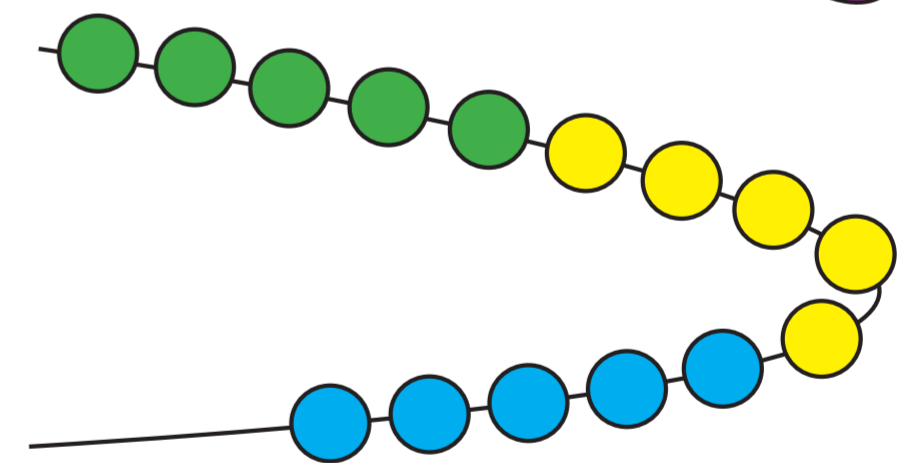
C: Concrete
(Real objects that children can touch and manipulate.)

C: Generic Concrete
(Physical resources that children can touch and manipulate which can represent a number or any everyday object.)

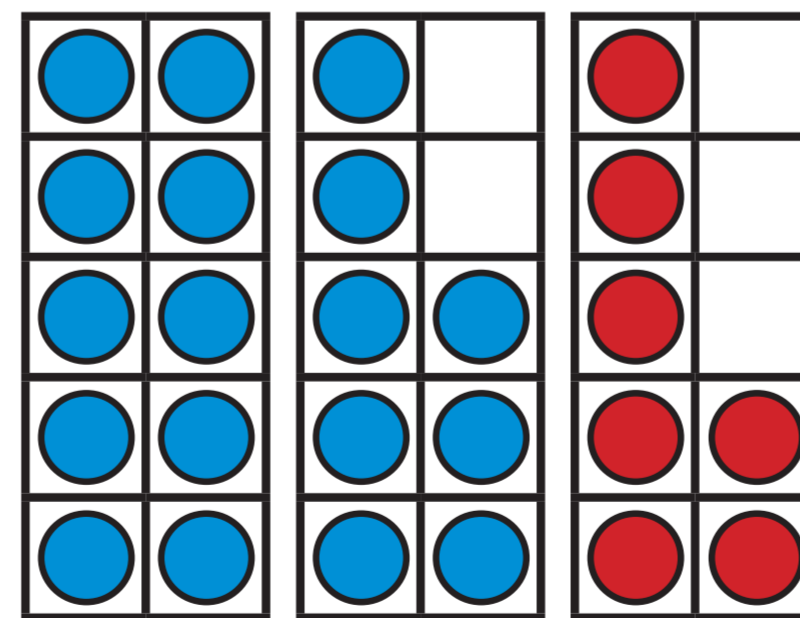


Cubes

Bead String

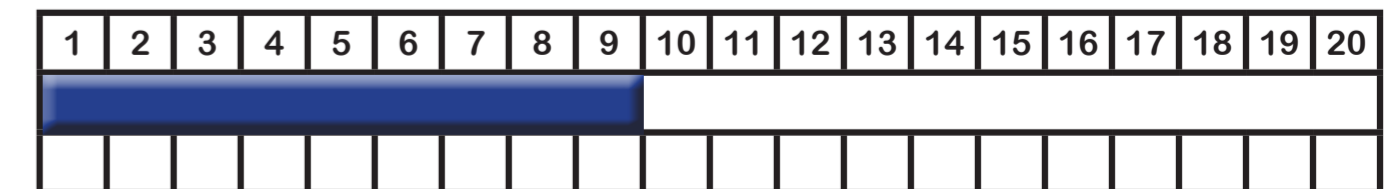


Number Shapes



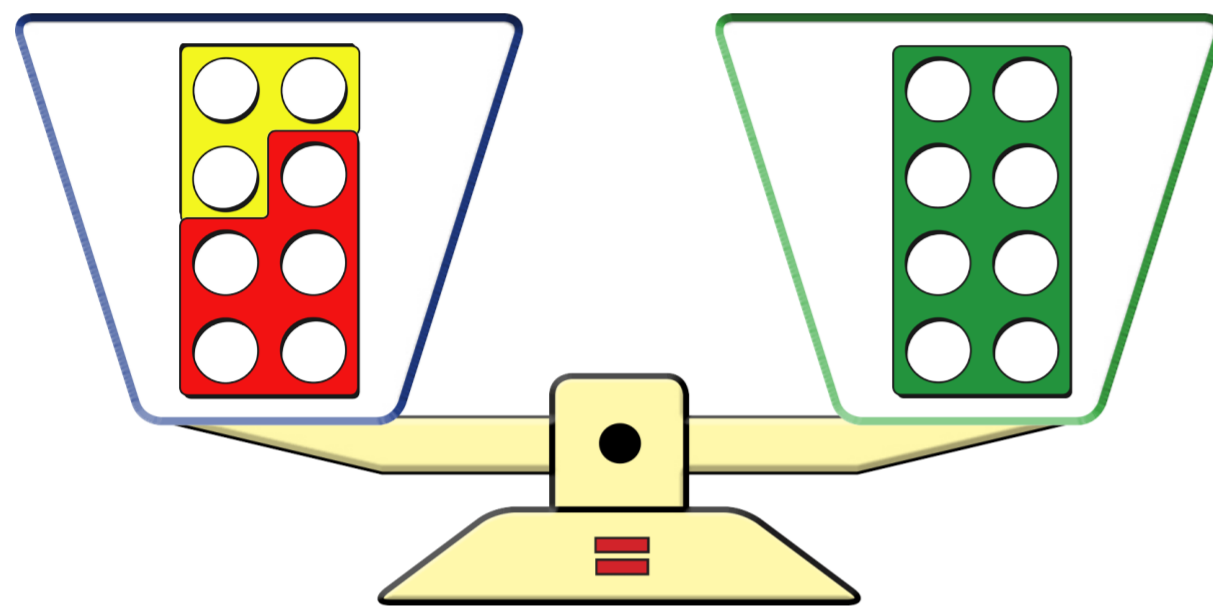
10s Frames
(Egg Boxes)

Number Rods

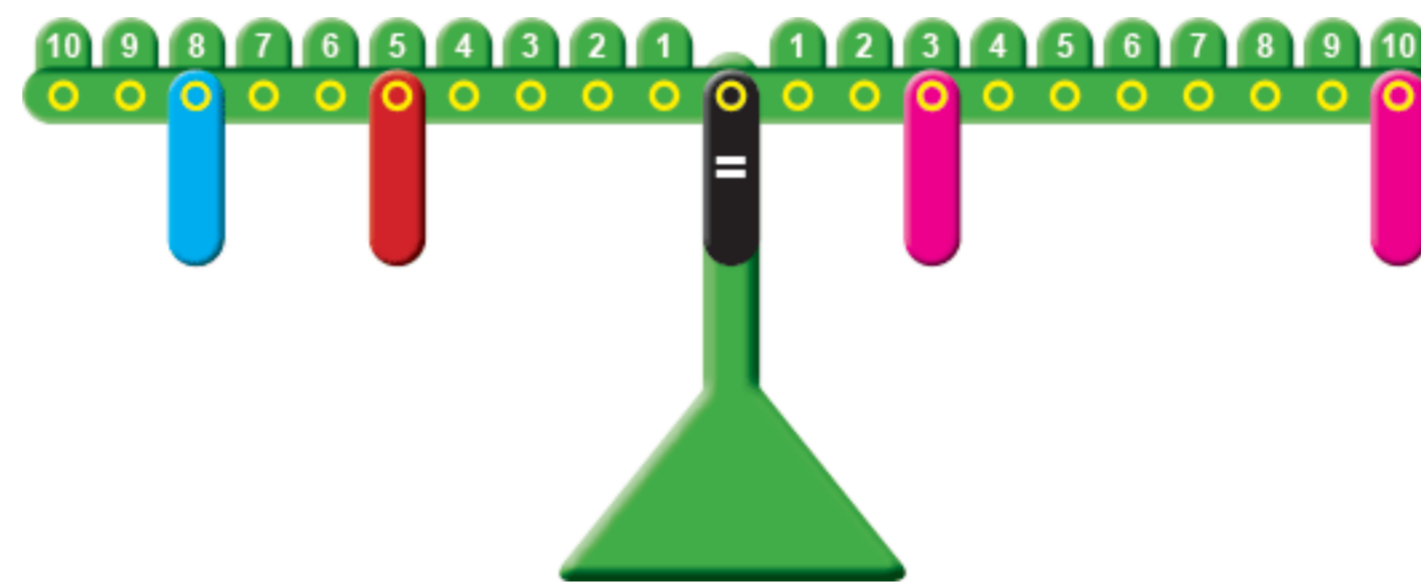




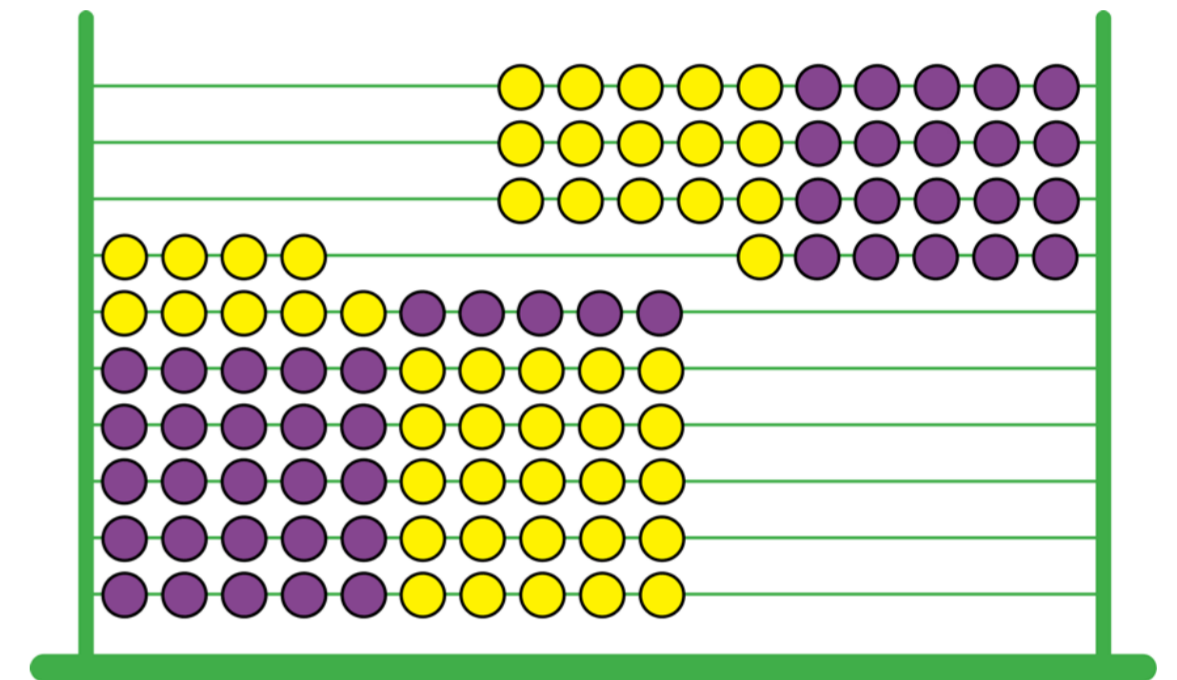
Pan Balance



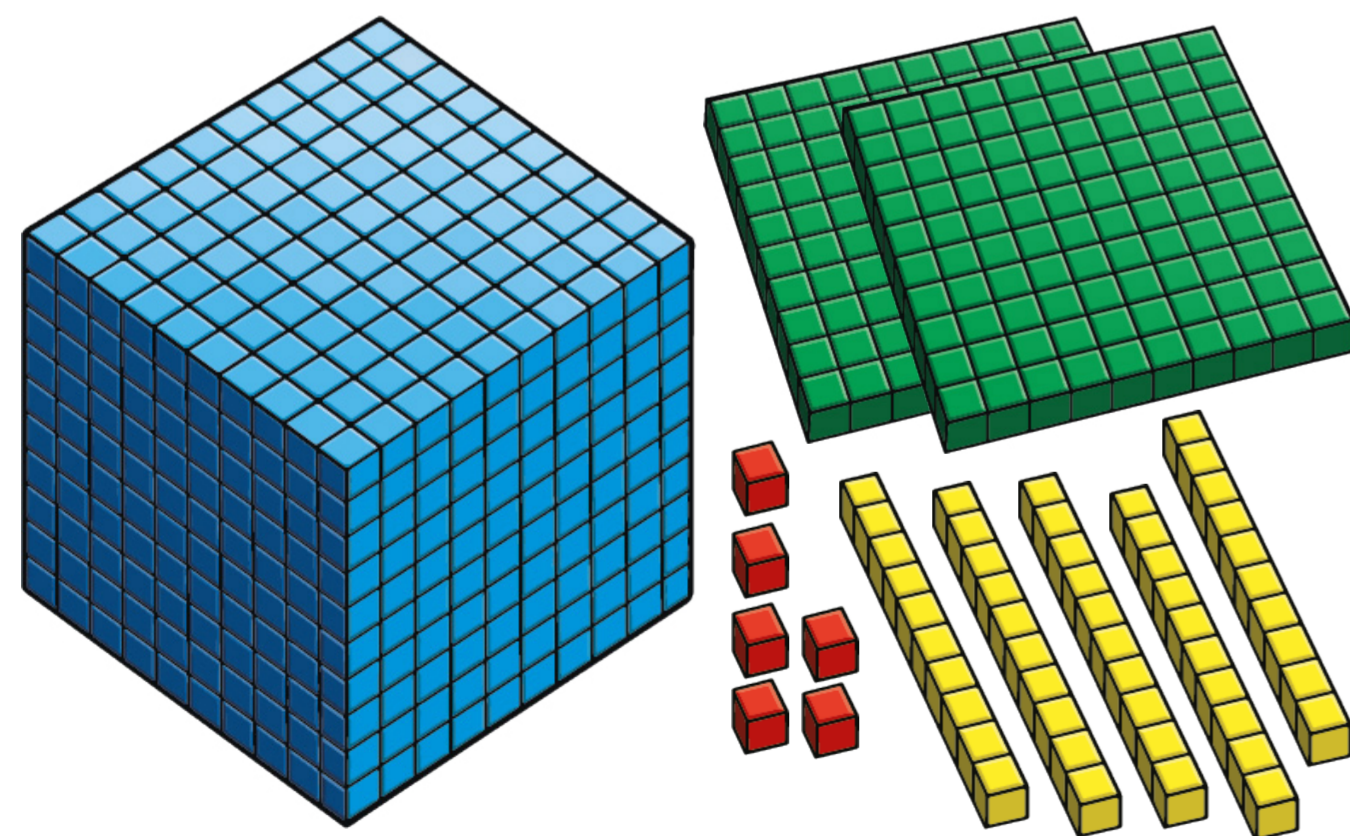
Balance Scales



Slavonic Abacus

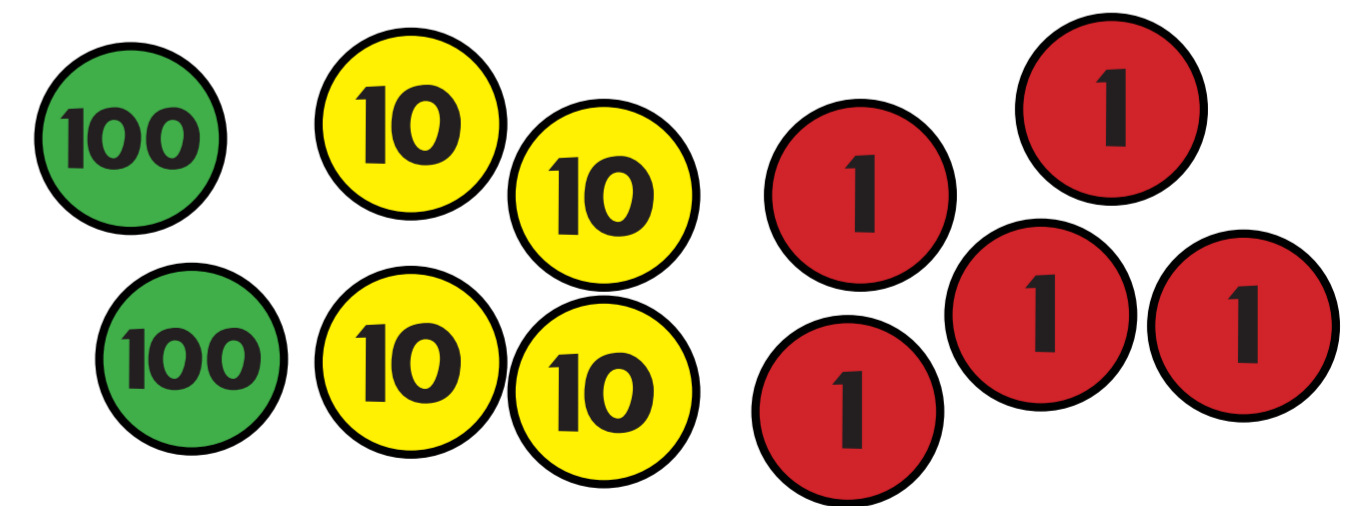


Base 10



C: Generic Concrete
(Physical resources that children can touch and manipulate which can represent a number or any everyday object.)

Place Value Counters

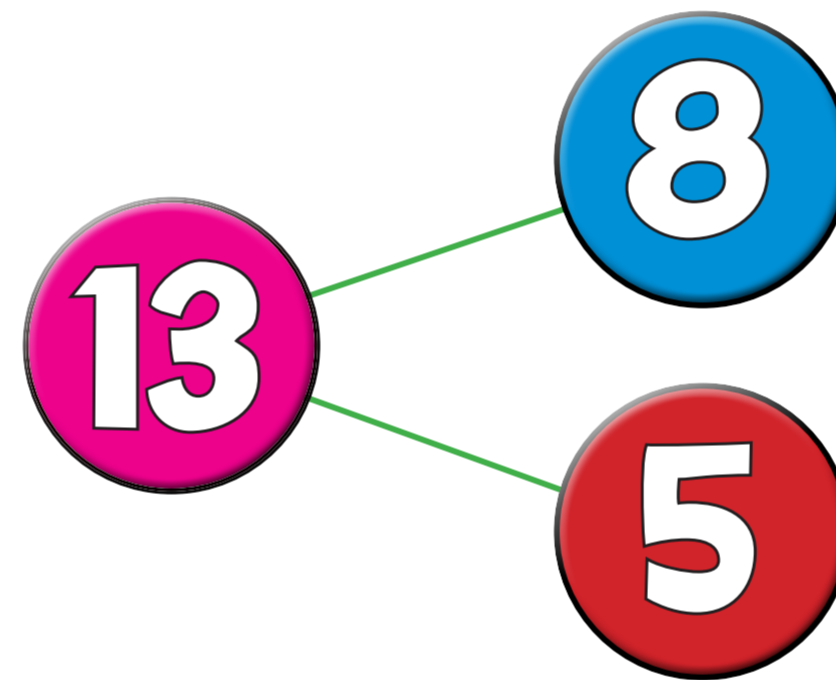




100 Square

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

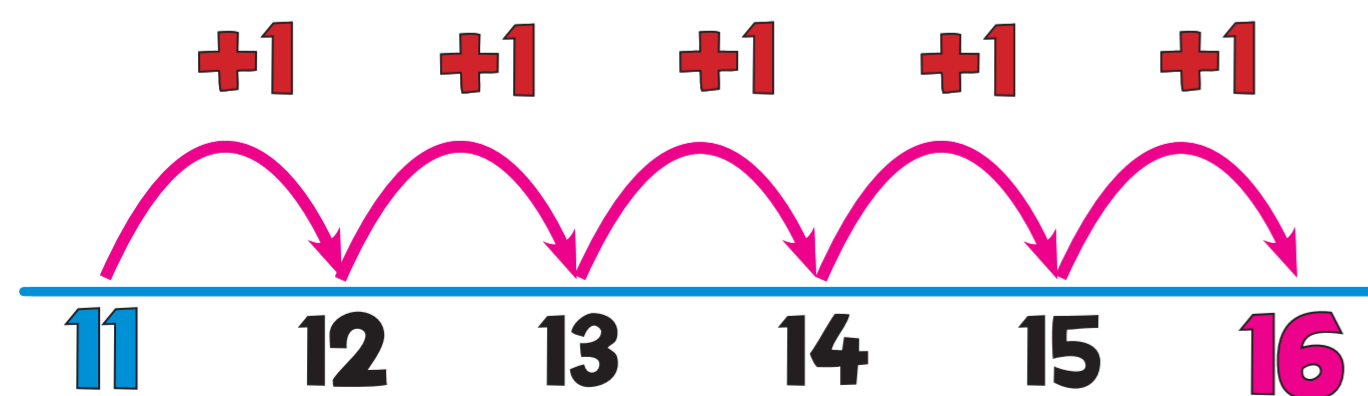
Number Bond Diagram



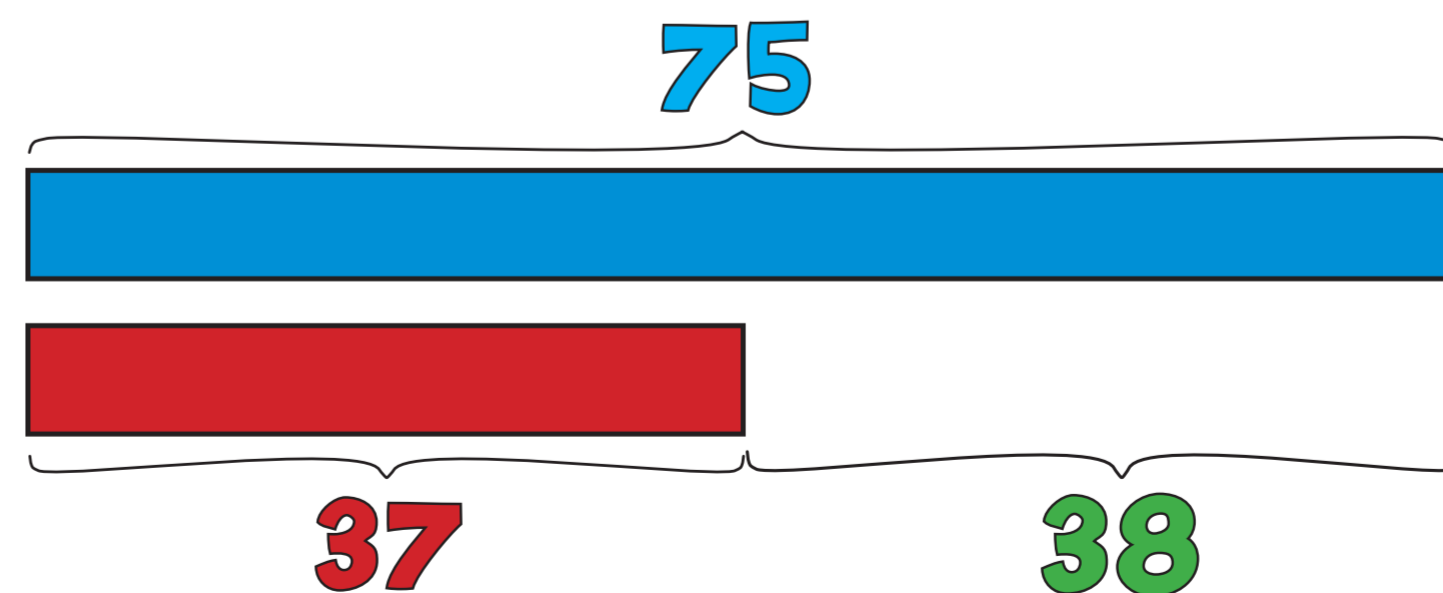
P: Pictorial
(Additional images that help children picture the mathematics.)



Number Line



Bar Model



CPA

reasoning

Addition Overview

CPA Models of Addition

Aggregation
Combining two sets of objects (counting all method)

Augmentation
Adding to a set (counting on method)

12

CPA reasoning $5 + 3 = 8$

13

CPA reasoning $8 + 5 = 13$

14

CPA reasoning $18 + 7 = 25$

15

CPA reasoning $57 + 6 = 63$

16

CPA reasoning $43 + 24 = 67$ Part 1

17

CPA reasoning $43 + 24 = 67$ Part 2

18

CPA reasoning $57 + 25 = 82$ Part 1

19

CPA reasoning $57 + 25 = 82$ Part 2

20

CPA reasoning $75 + 52 = 127$ Part 1

21

CPA reasoning $75 + 52 = 127$ Part 2

22

CPA reasoning $75 + 52 = 127$ Part 3

23

CPA reasoning $86 + 48 = 134$ Part 1

24

CPA reasoning $86 + 48 = 134$ Part 2

25

CPA reasoning $86 + 48 = 134$ Part 3

26

CPA reasoning $687 + 248 = 935$

27

CPA reasoning $738 + 524 = 1262$

28

CPA reasoning $4.8 + 3.8 = 8.6$

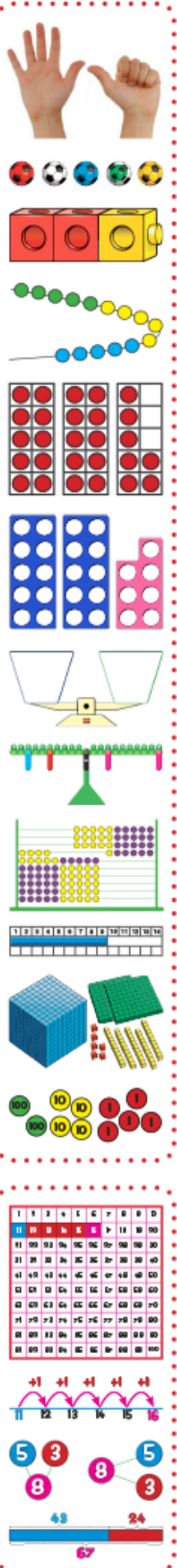
29

CPA reasoning $5.65 + 3.29 = 8.94$

30

CPA reasoning $76.7 + 58.5 = 135.2$

31



CPA

reasoning

Subtraction Overview

CPA Models of Subtraction

Removing Items
Take Away
"If I had 8 footballs and kicked 3 over the fence, how many did I have left?"
 $8 - 3 = 5$

Comparing Sets
Comparison
"If I had 9 footballs and you had 6, how many more balls have I got than you?"
 $9 - 6 = 3$

Reduction
"The football cost £8 but I got £3 off in the sale. How much did I pay?"
 $8 - 3 = 5$

Inverse of Addition
"The football costs £8 but I've only got £5. How much more do I need?"
 $6 + 1 = 7$

Whole / Part
"There are 10 footballs in my bag. 6 are red, how many are yellow?"
 $10 - 6 = 4$

CPA reasoning $7 - 3 = 4$

Count Back Images

CPA reasoning $7 - 5 = 2$

Count On Images

CPA reasoning $12 - 3 = 9$

Count Back Images

CPA reasoning $12 - 9 = 3$

Count On Images

CPA reasoning $87 - 23 = 64$

CPA reasoning $87 - 23 = 64$

CPA reasoning $75 - 37 = 38$

CPA reasoning $75 - 37 = 38$

CPA reasoning $127 - 74 = 53$

CPA reasoning $127 - 74 = 53$

CPA reasoning $127 - 74 = 53$

CPA reasoning $132 - 56 = 76$

CPA reasoning $132 - 56 = 76$

CPA reasoning $132 - 56 = 76$

CPA reasoning $736 - 462 = 274$

CPA reasoning $723 - 356 = 367$

CPA reasoning $723 - 356 = 367$

CPA reasoning $1375 - 538 = 837$

CPA reasoning $1375 - 538 = 837$

It's a 'take away' image!

CPA reasoning $5042 - 1776 = 3266$

CPA reasoning $5042 - 1776 = 3266$

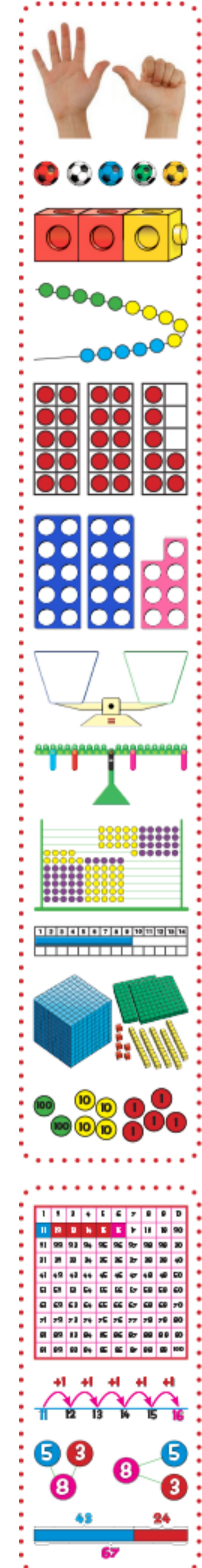
CPA reasoning $13.4 - 8.7 = 4.7$

CPA reasoning $72.43 - 47.85 = 24.58$

CPA reasoning $72.43 - 47.85 = 24.58$

CPA reasoning $12.4 - 5.97 = 6.43$

CPA reasoning $12.4 - 5.97 = 6.43$



CPA

reasoning

Multiplication Overview

CPA Models of Multiplication

Repeated Addition

5 + 5 + 5 = 15

"I can fit 5 footballs in a bag and I've brought 3 full bags. How many footballs have with me? "15"

Scaling

"My tower of football stickers was 5 high, but now it's 3 times as big! How many stickers in my tower?" "15"

$5 \times 3 = 15$

59

CPA reasoning

$5 \times 3 = 15$

5 + 5 + 5 = 15

"5 times 3"

60

CPA reasoning

$7 \times 4 = 28$

$7 + 7 + 7 + 7 = 28$

"7 times 4"

61

CPA reasoning

$8 \times 9 = 72$

$8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 = 72$

"8 multiplied by 9"

62

CPA reasoning

$15 \times 5 = 75$

$10 \times 5 = 50$ $5 \times 5 = 25$

$50 + 25 = 75$

63

CPA reasoning

$43 \times 6 = 258$

$40 \times 6 = 240$ $3 \times 6 = 18$

$240 + 18 = 258$

64

CPA reasoning

$147 \times 4 = 588$

$100 \times 4 = 400$ $40 \times 4 = 160$ $7 \times 4 = 28$

$400 + 160 + 28 = 588$

65

CPA reasoning

$15 \times 12 = 180$

$100 + 50 + 20 + 10 = 180$

66

CPA reasoning

$3.6 \times 4 = 14.4$

$3 \times 4 = 12$ $0.6 \times 4 = 2.4$

$12 + 2.4 = 14.4$

67

CPA reasoning

$47.2 \times 3 = 141.6$

$40 \times 3 = 120$ $7 \times 3 = 21$ $0.2 \times 3 = 0.6$

$120 + 21 + 0.6 = 141.6$

68

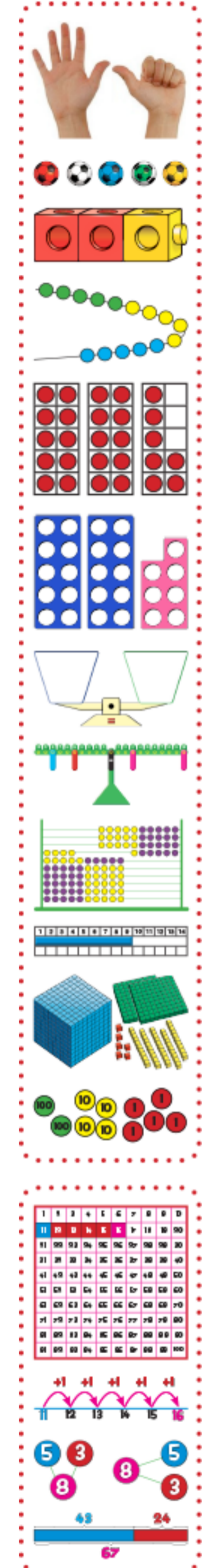
CPA reasoning

$7.38 \times 6 = 44.28$

$7 \times 6 = 42$ $0.3 \times 6 = 1.8$ $0.08 \times 6 = 0.48$

$42 + 1.8 + 0.48 = 44.28$

69



Division Overview

CPA reasoning Models of Division

Sharing
 5 in each group!
 15 ÷ 3 = 5
 "If I shared my 15 footballs fairly into 3 bags, how many balls would be in each bag?" "5"

Grouping
 5 groups
 15 ÷ 3 = 5
 "If I can put my 15 footballs into groups of 3, how many groups would I create?" "5"

70

CPA reasoning 12 ÷ 2 = 6 Part 1

Sharing Model
 6 footballs in each goal!
 12

71

CPA reasoning 12 ÷ 2 = 6 Part 2

Grouping Model
 "12 divided by 2"
 "How many 2s in 12?"
 12 footballs in 6 groups of 2!

72

CPA reasoning 20 ÷ 5 = 4

"How many 5s in 20?"
 "20 divided by 5"

73

CPA reasoning 23 ÷ 5 = 4r3

"How many 5s in 23?"
 "23 divided by 5"

74

CPA reasoning 48 ÷ 8 = 6

"How many 8s in 48?"
 "48 divided by 8"

75

CPA reasoning 72 ÷ 4 = 18

76

CPA reasoning 65 ÷ 4 = 16r1

77

CPA reasoning 136 ÷ 4 = 34

78

CPA reasoning 145 ÷ 6 = 24r1

79

CPA reasoning 394 ÷ 6 = 65r4

80

CPA reasoning 536 ÷ 4 = 134

81

CPA reasoning 1278 ÷ 6 = 213

82

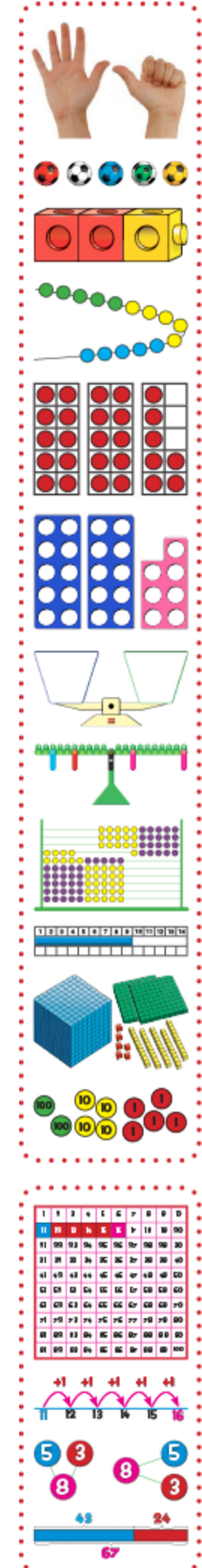
CPA reasoning 18 ÷ 1.5 = 12

"How many 1.5s are there in 18?"

83

CPA reasoning 87.5 ÷ 7 = 12.5

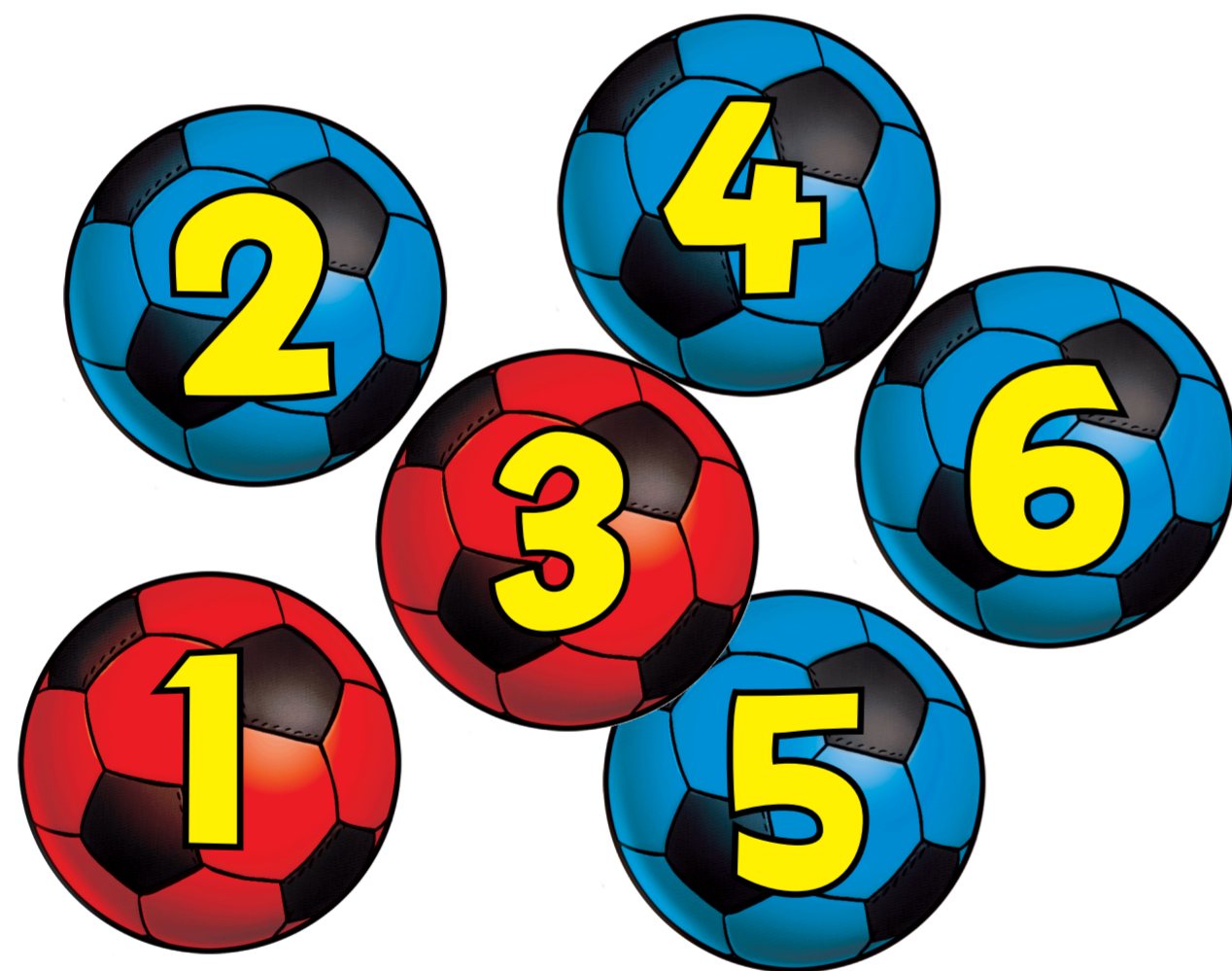
84





Aggregation

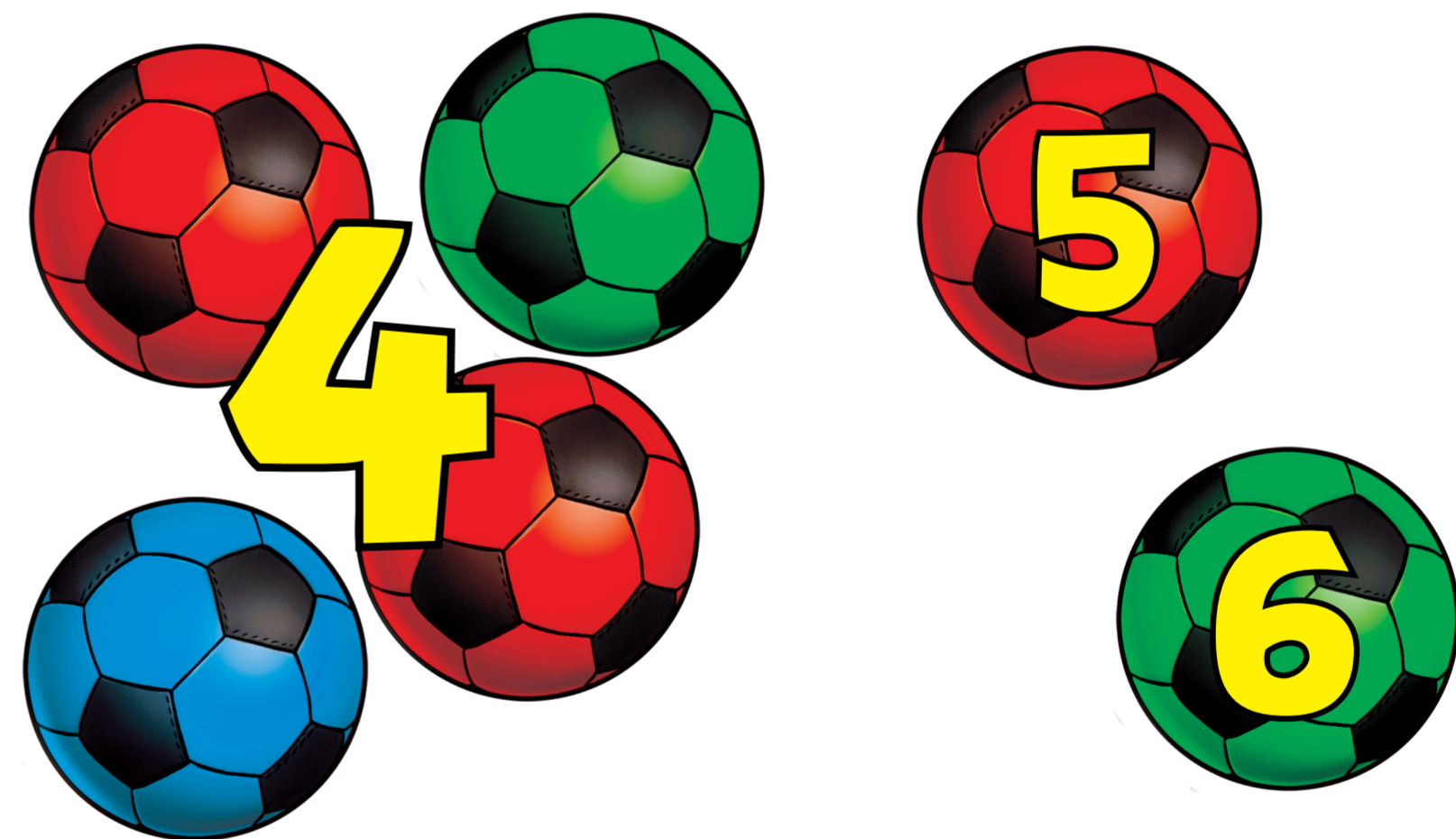
Combining two sets of objects
(counting all method!)



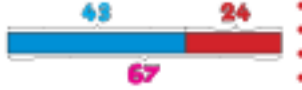
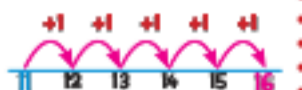
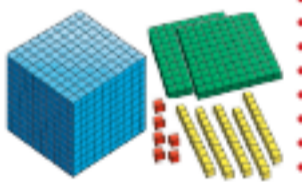
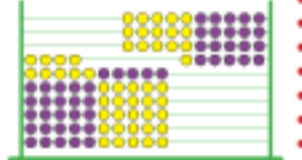
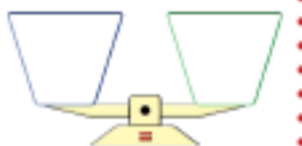
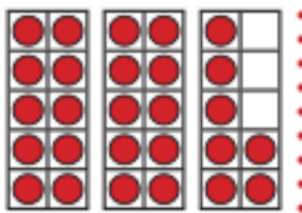
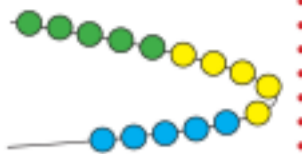
“If I have 4 blue footballs and 2 red footballs in my bag, how many footballs do I have all together?” “6”

Augmentation

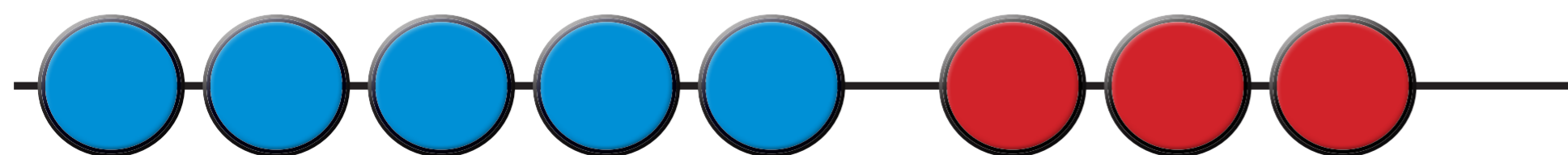
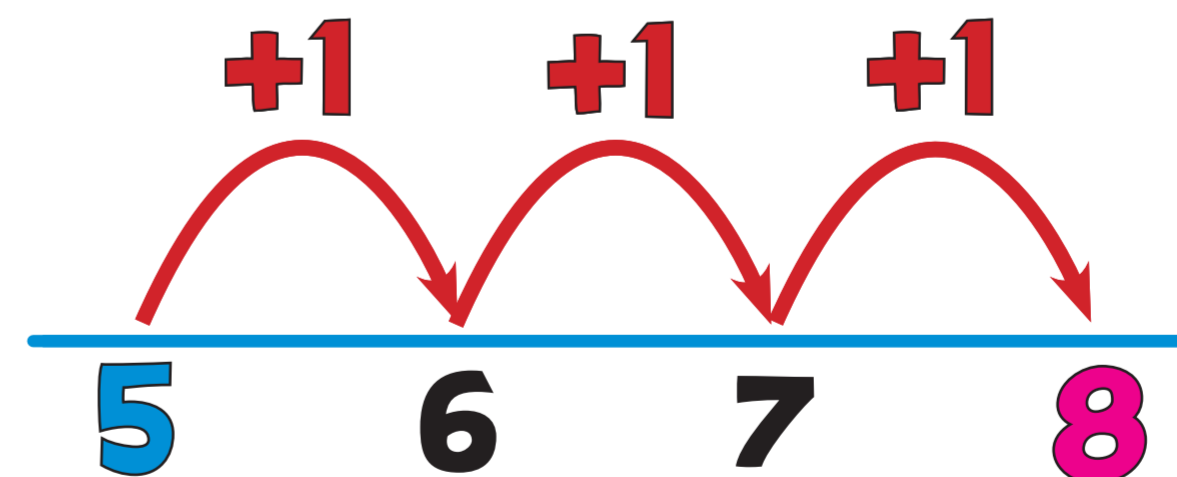
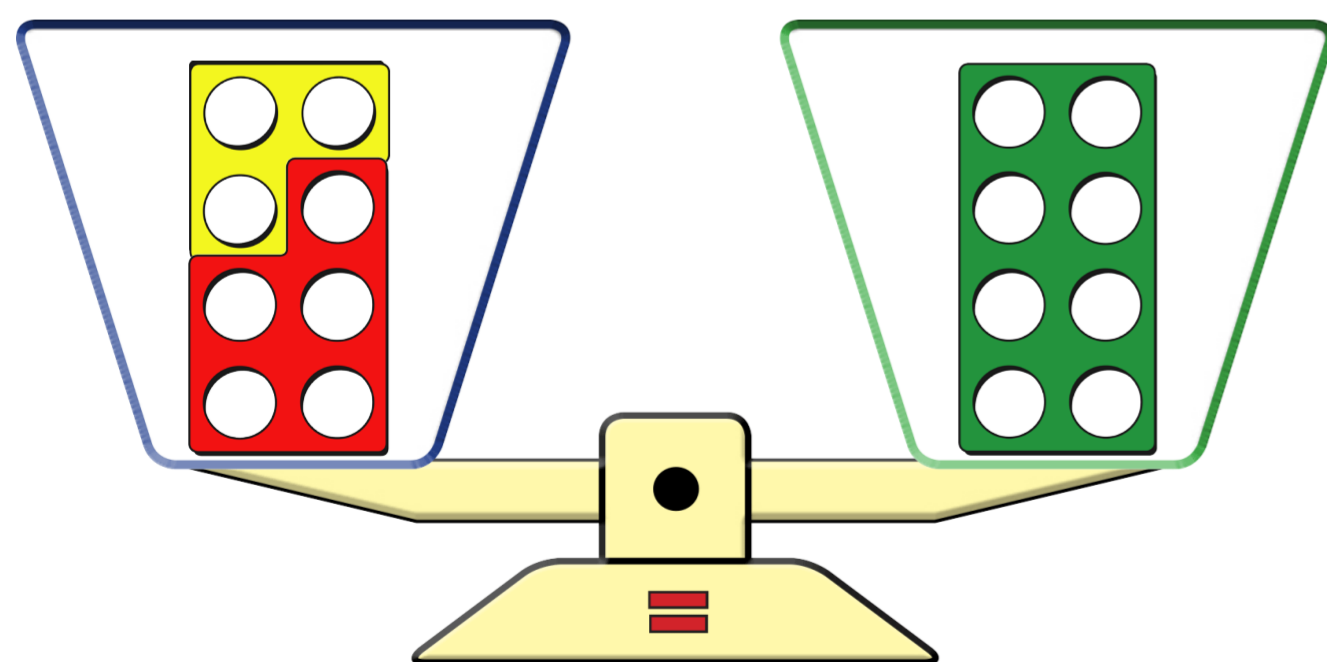
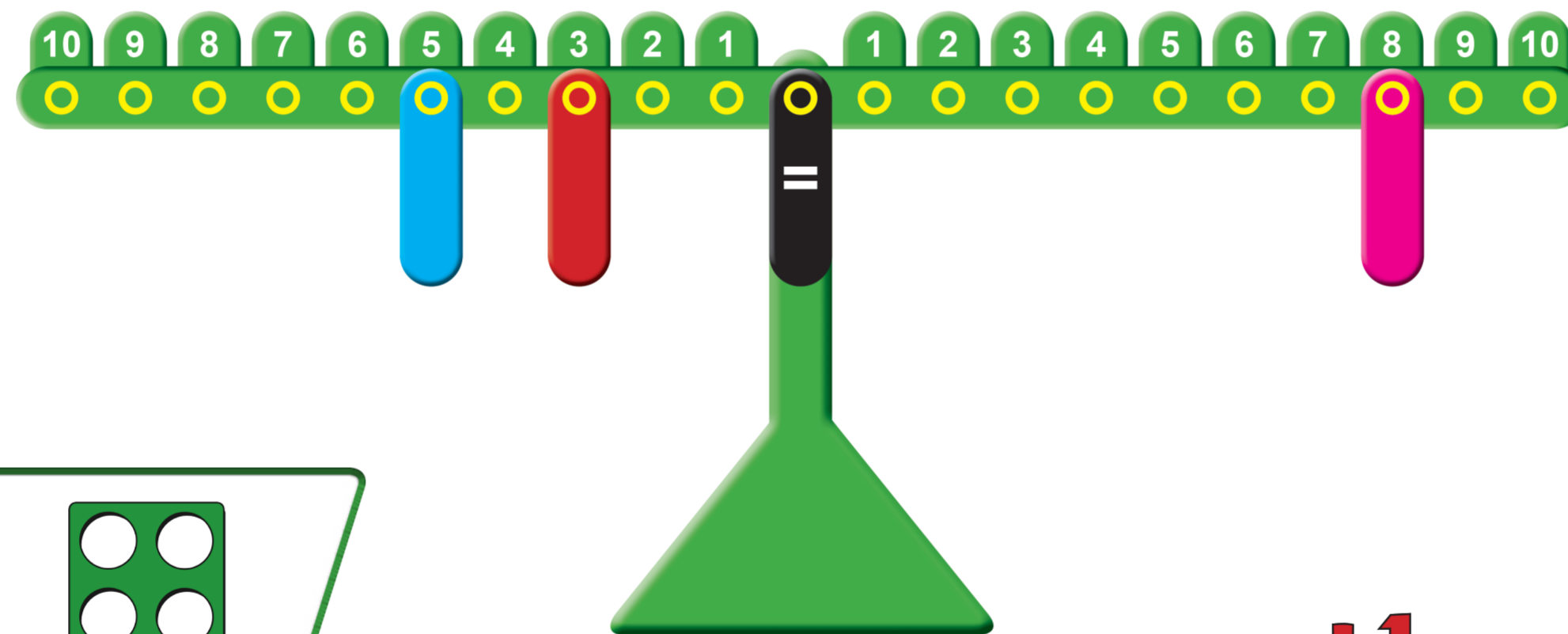
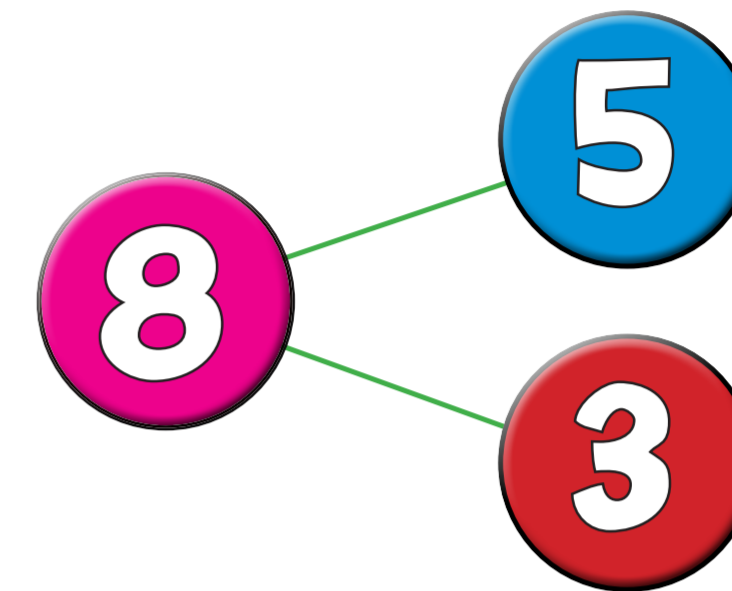
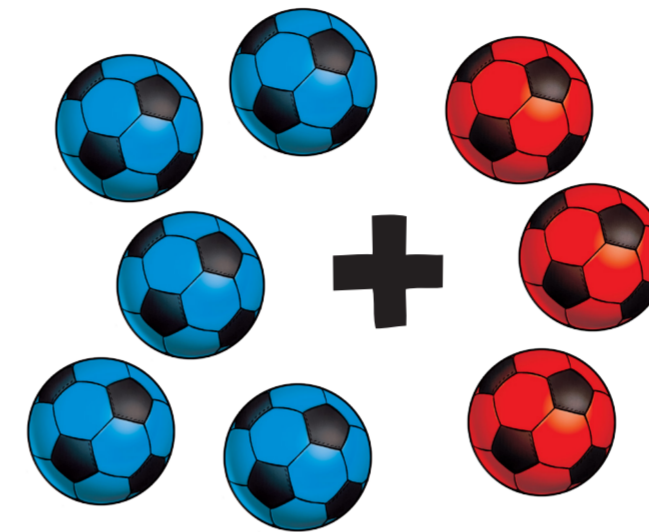
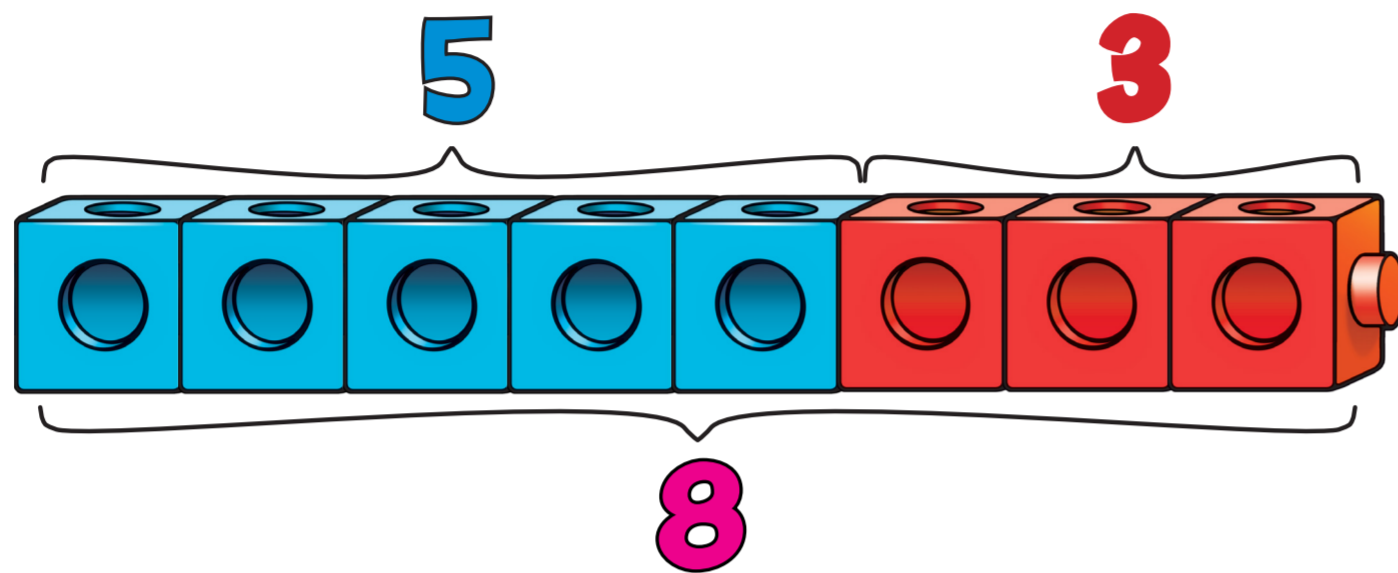
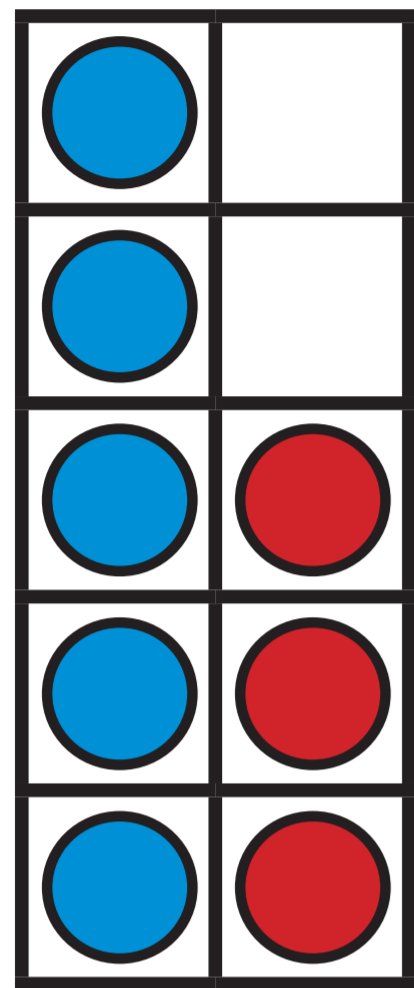
Adding to a set
(counting on method!)



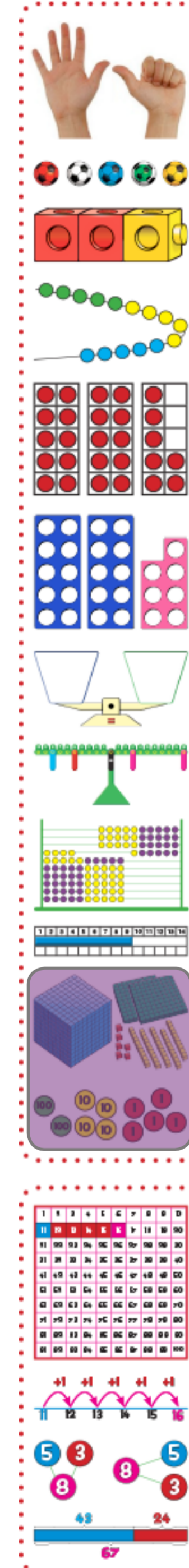
“If I had 4 footballs and then found 2 more, how many footballs would I have in total?” “6”



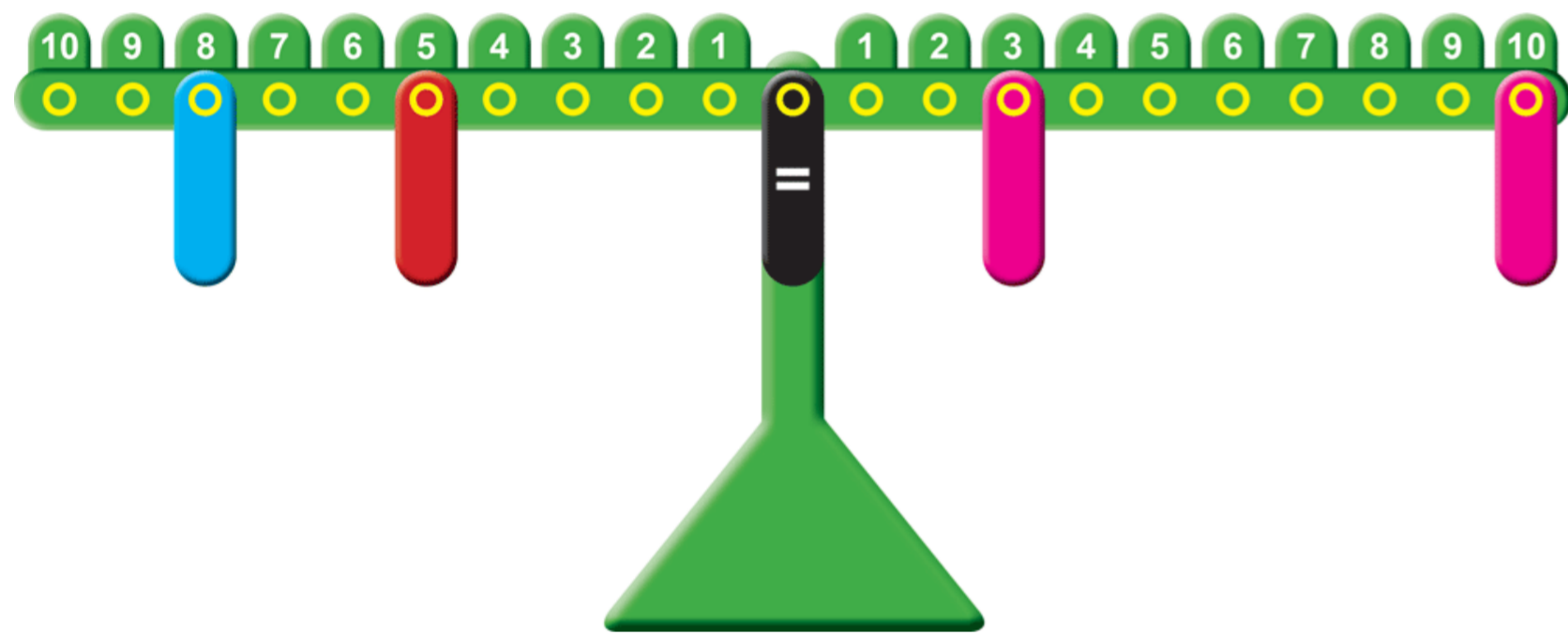
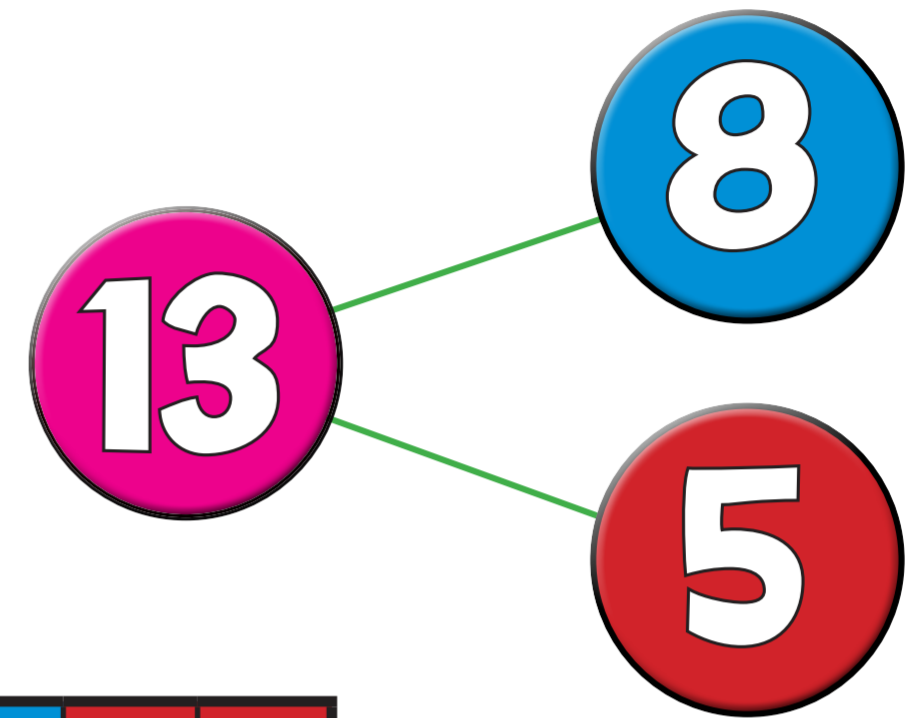
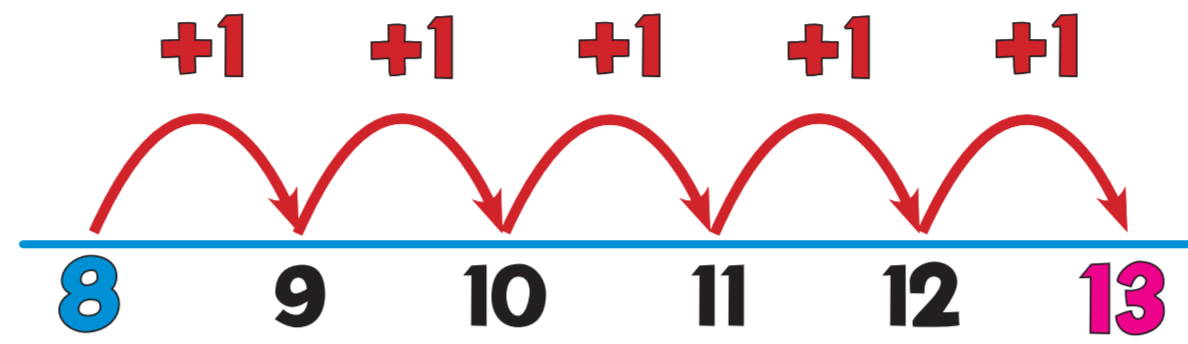
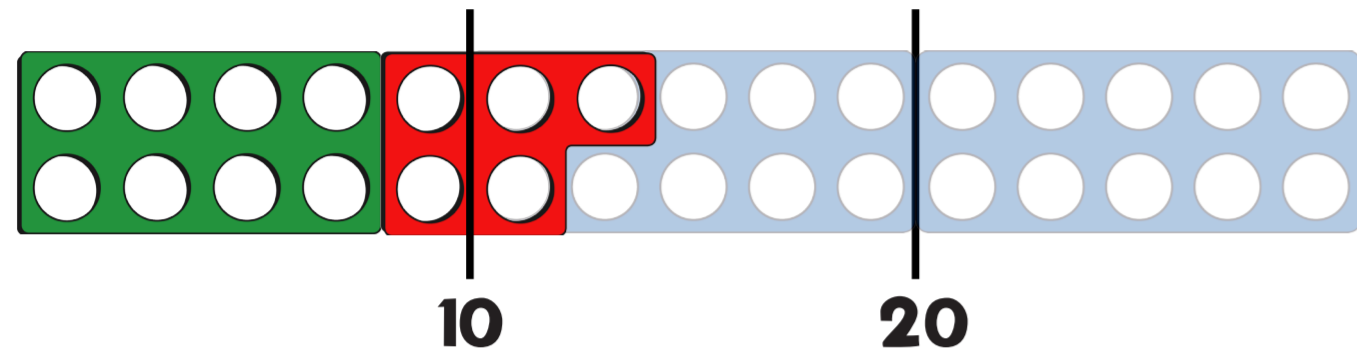
$$5 + 3 = 8$$



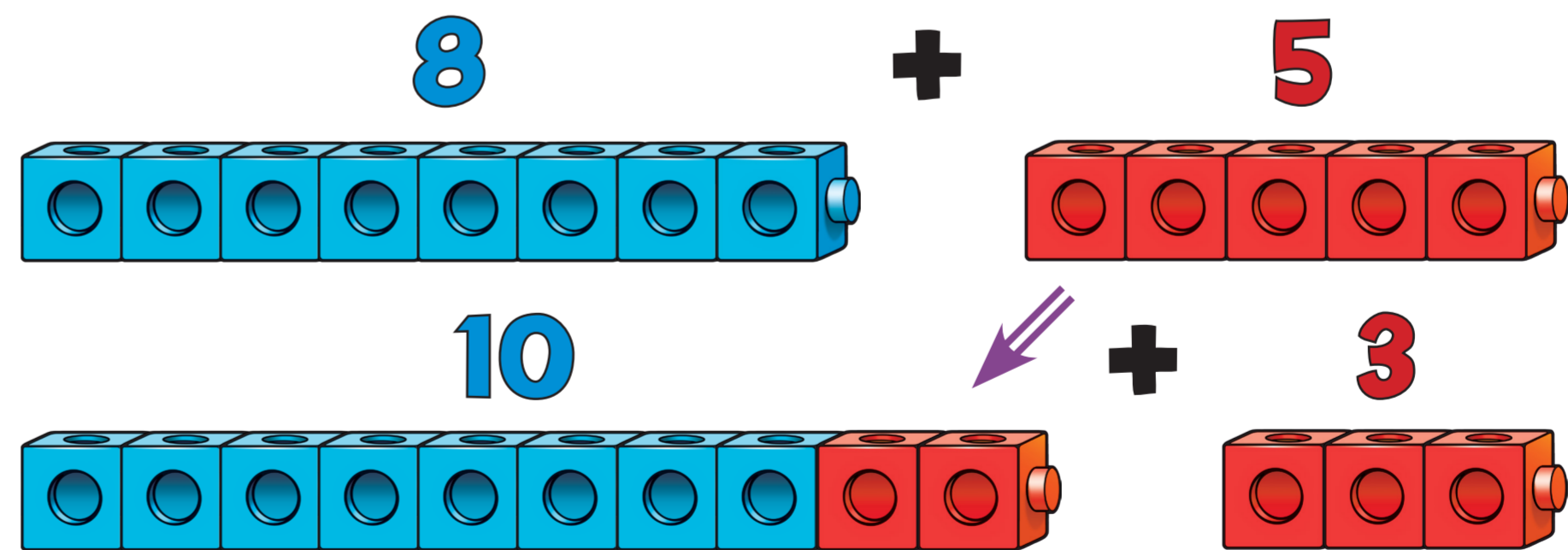
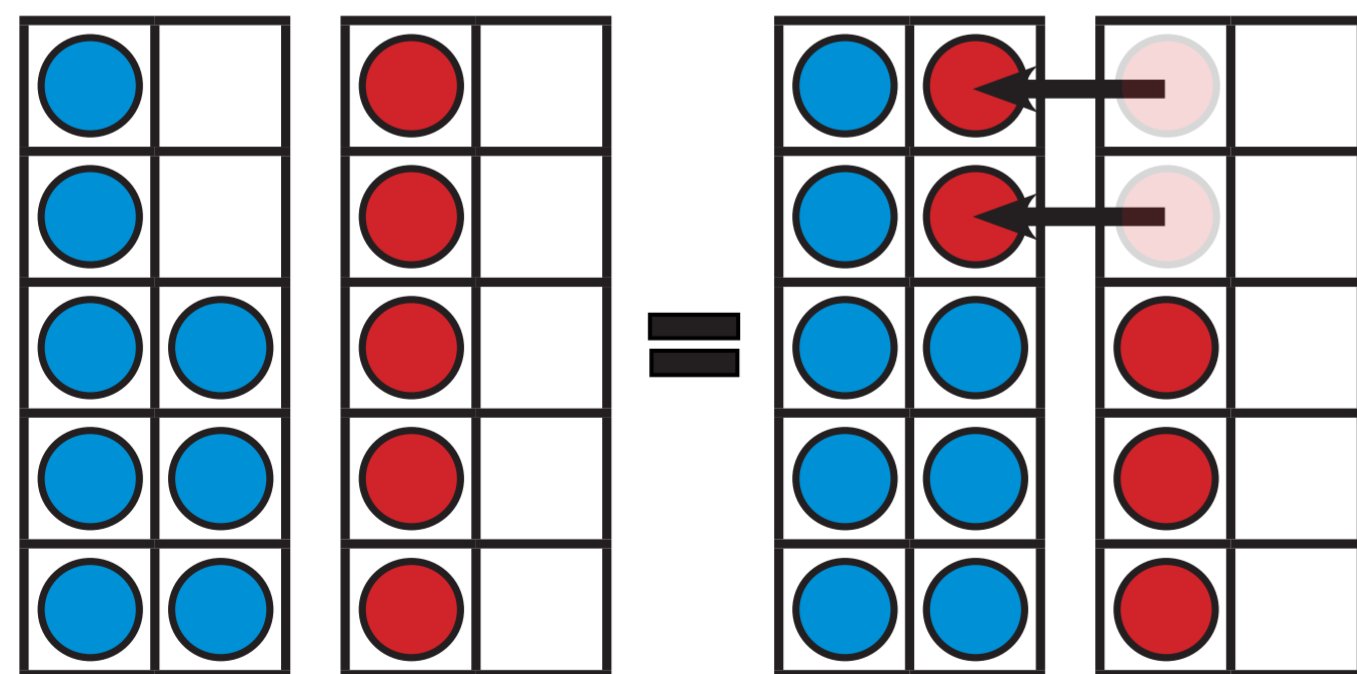
$$8 = 5 + 3$$



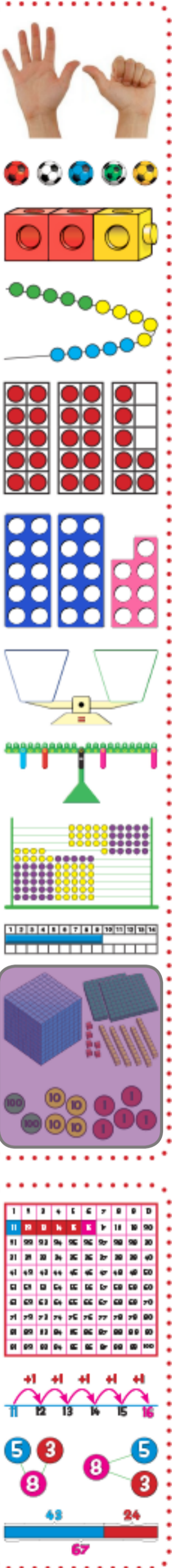
$$8 + 5 = 13$$



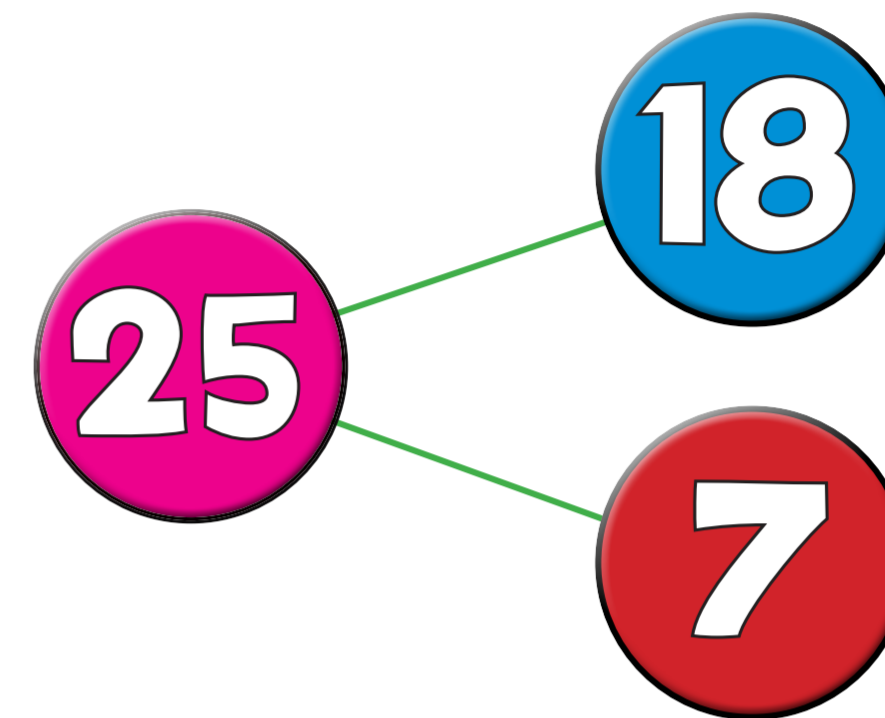
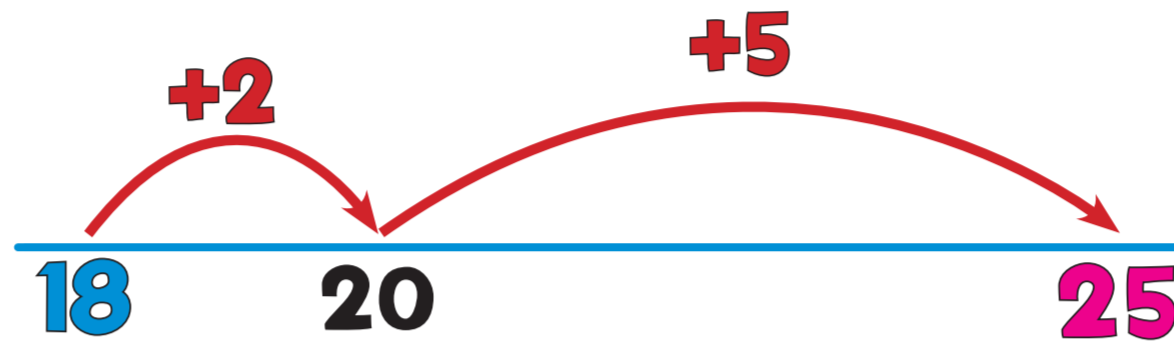
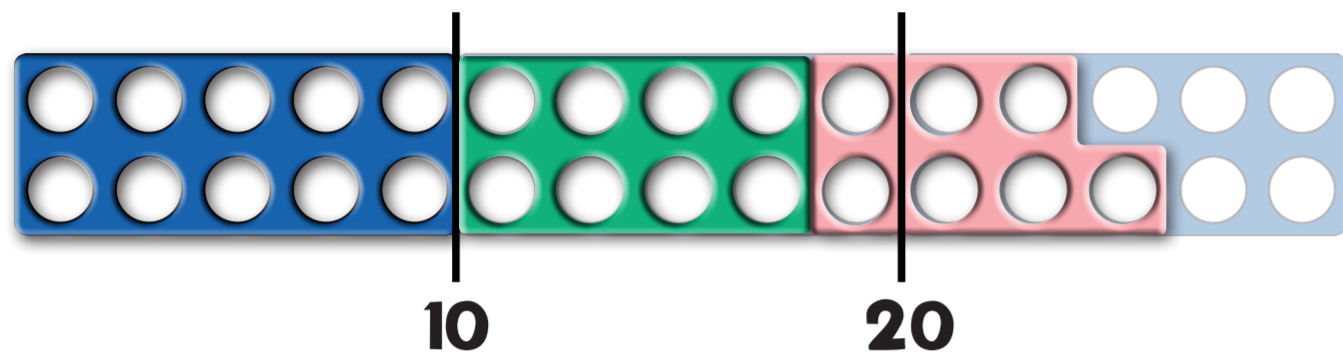
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30



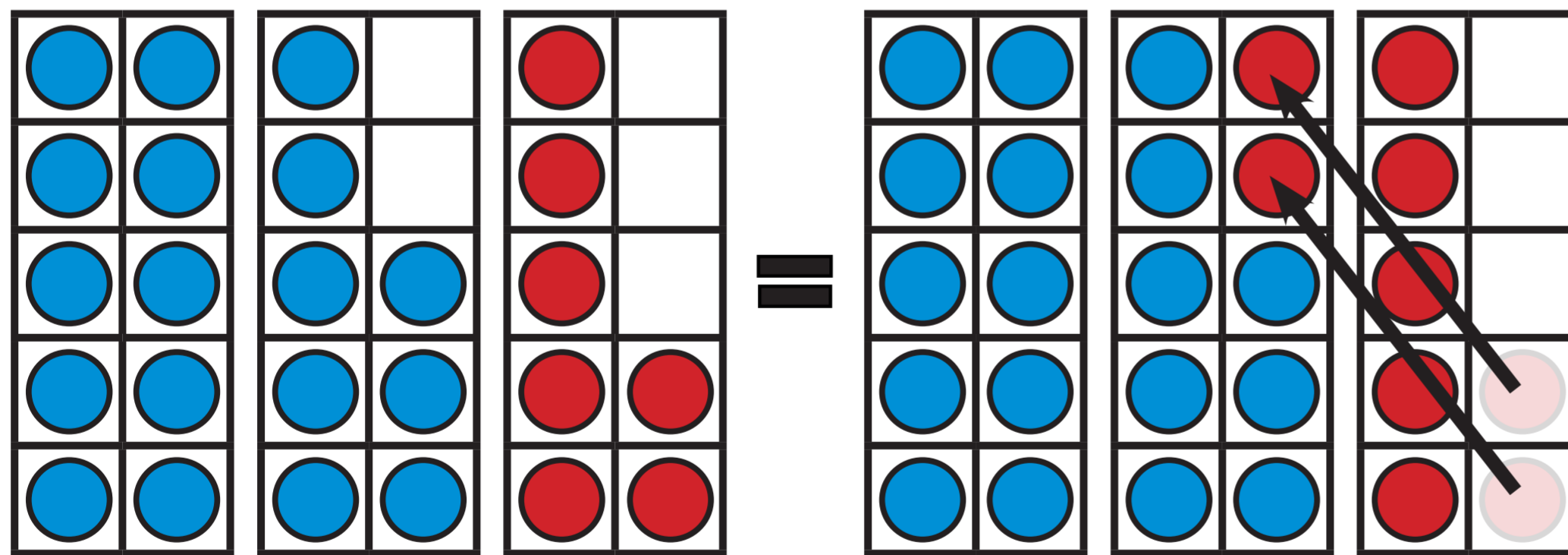
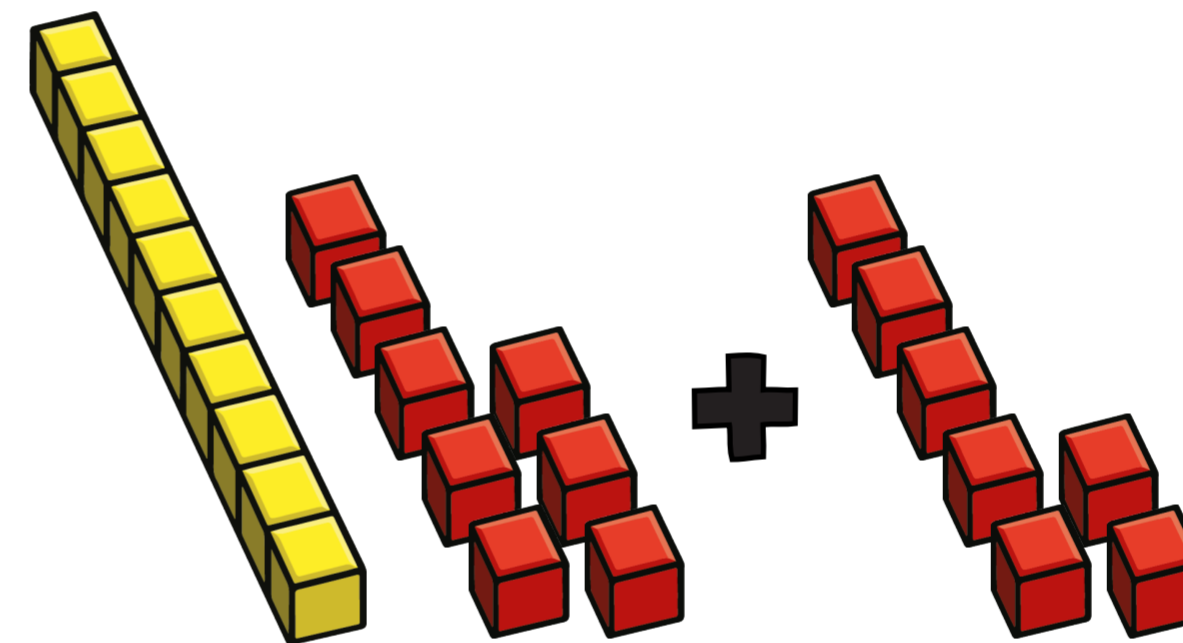
$$13 = 8 + 5$$



$$18 + 7 = 25$$

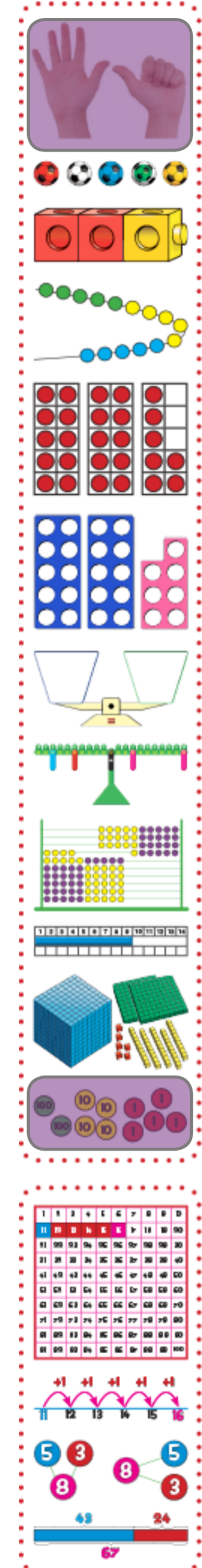


1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
41	42	43	44	45	46	47	48	49	50



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
																		[Shaded area from 19 to 30]											

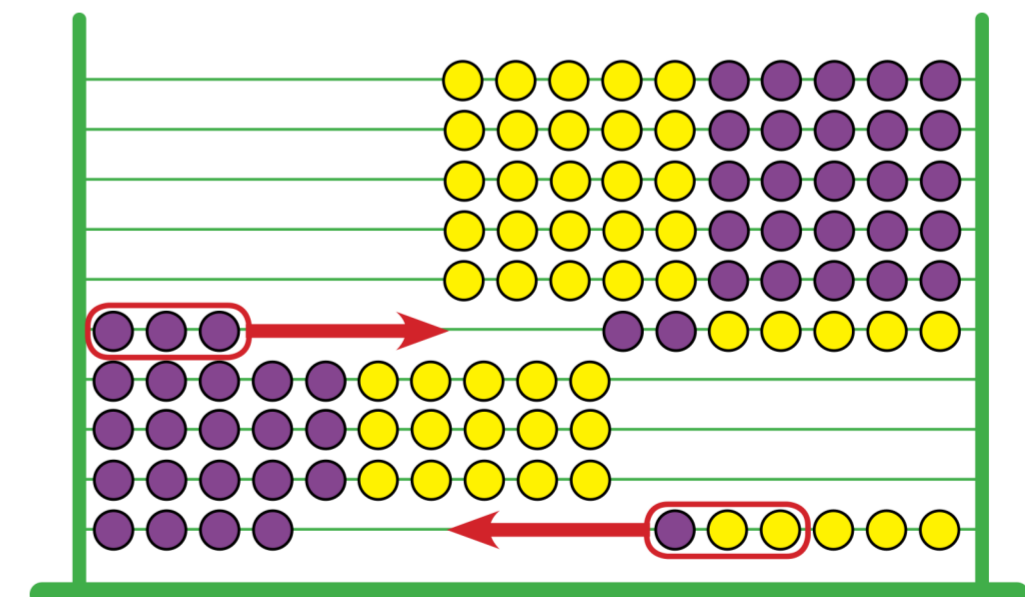
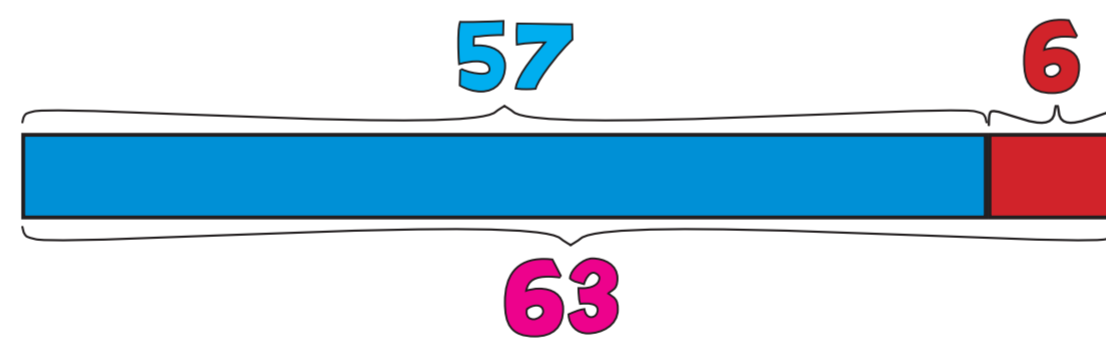
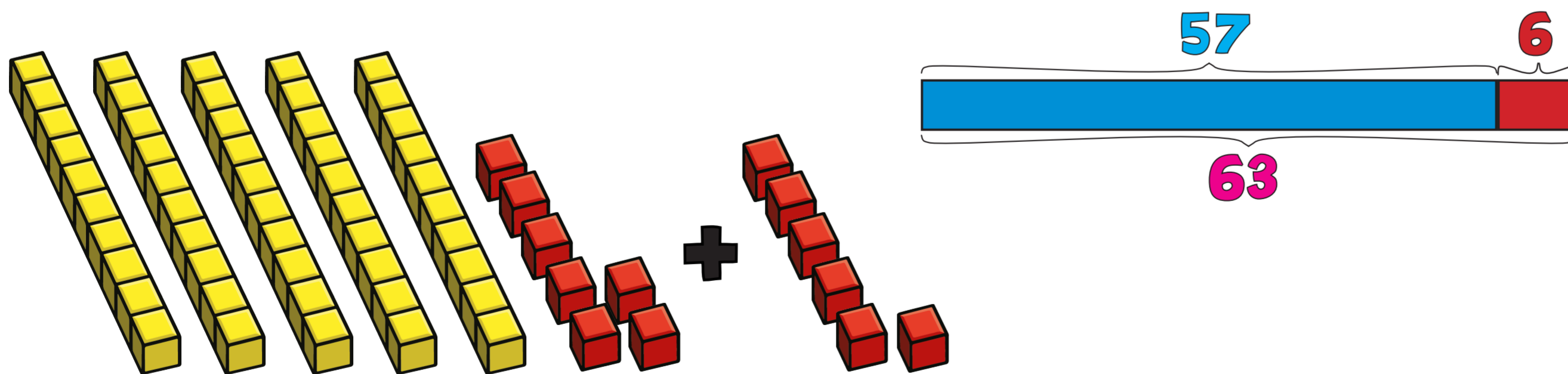
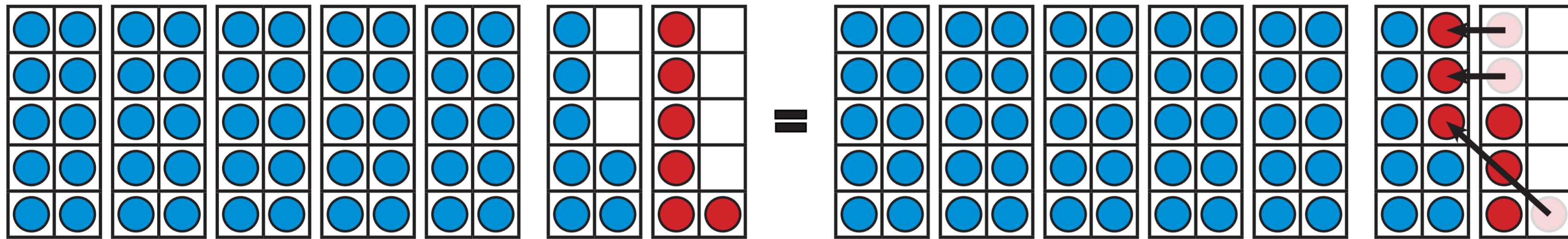
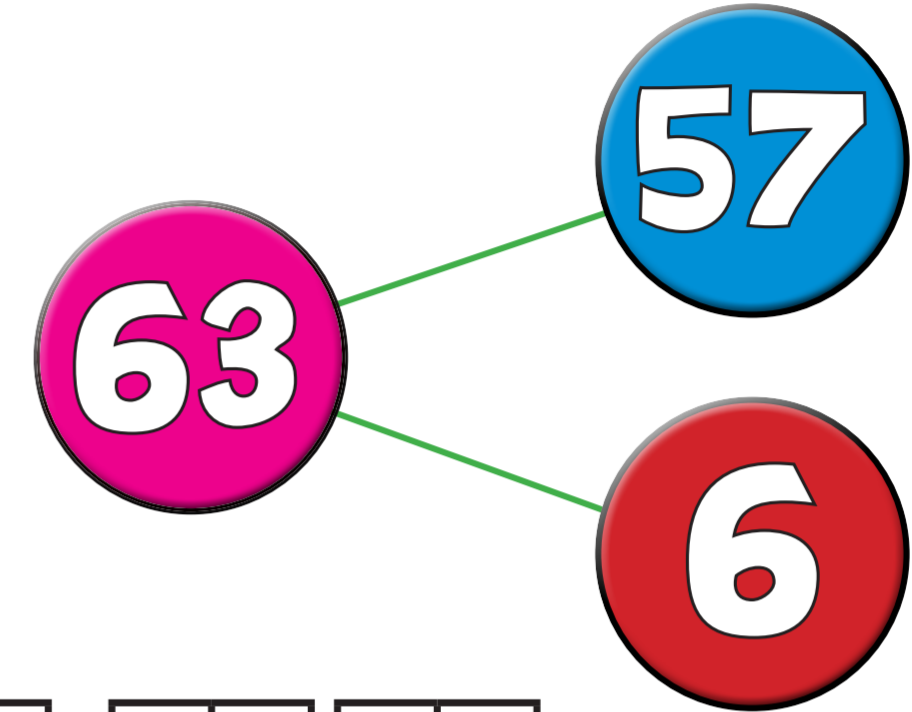
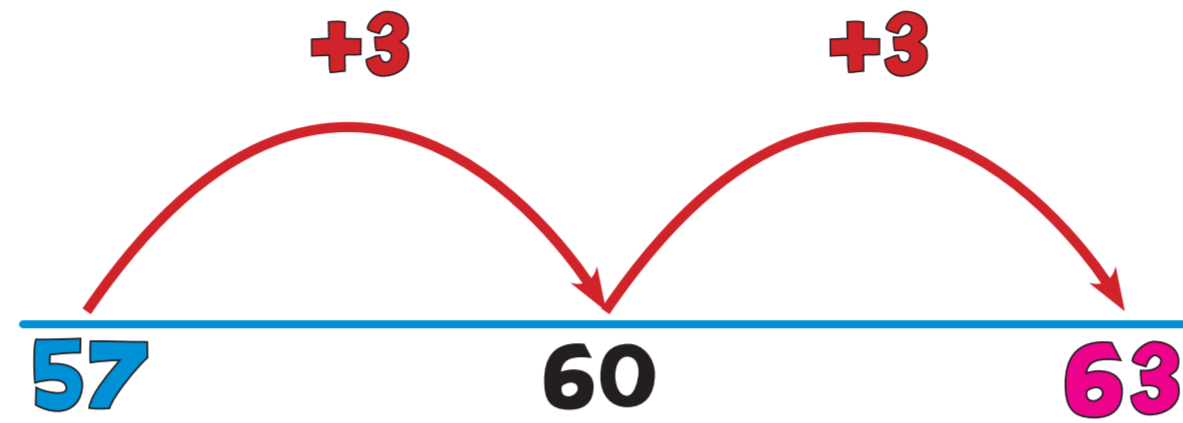
$$25 = 18 + 7$$



$$57 + 6 = 63$$

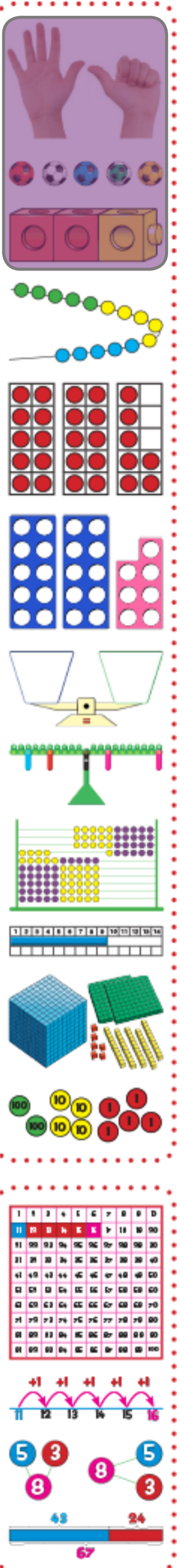


41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80

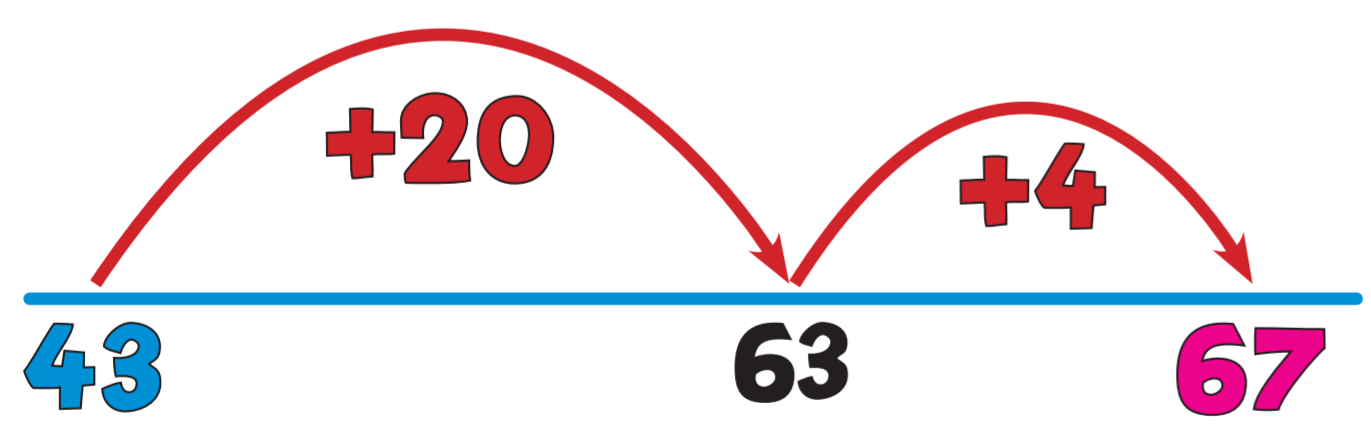
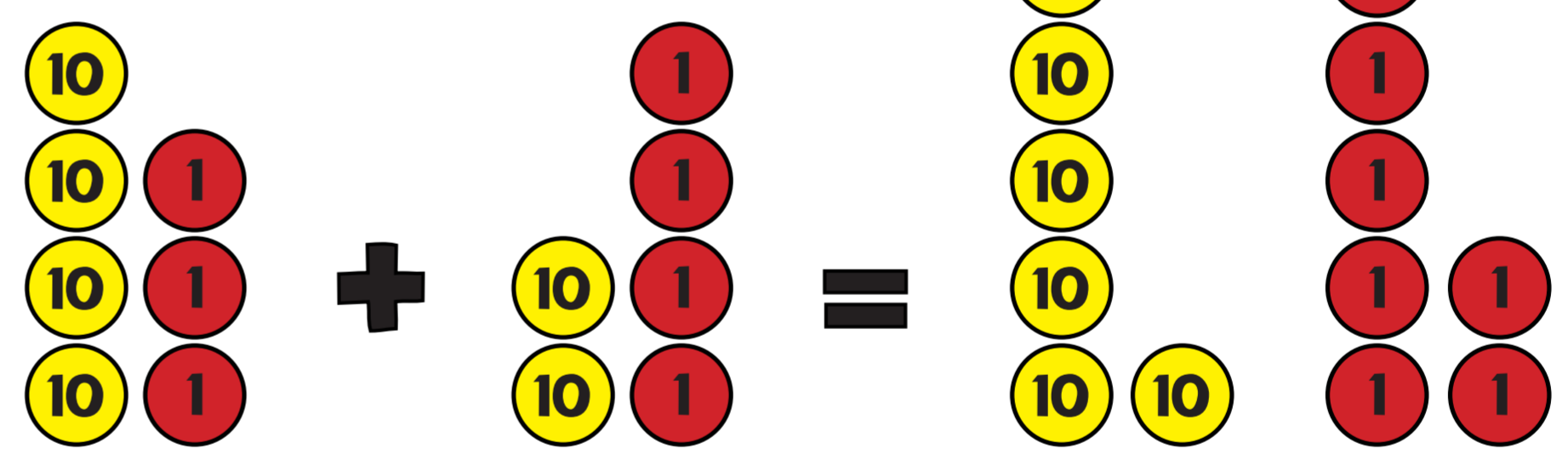
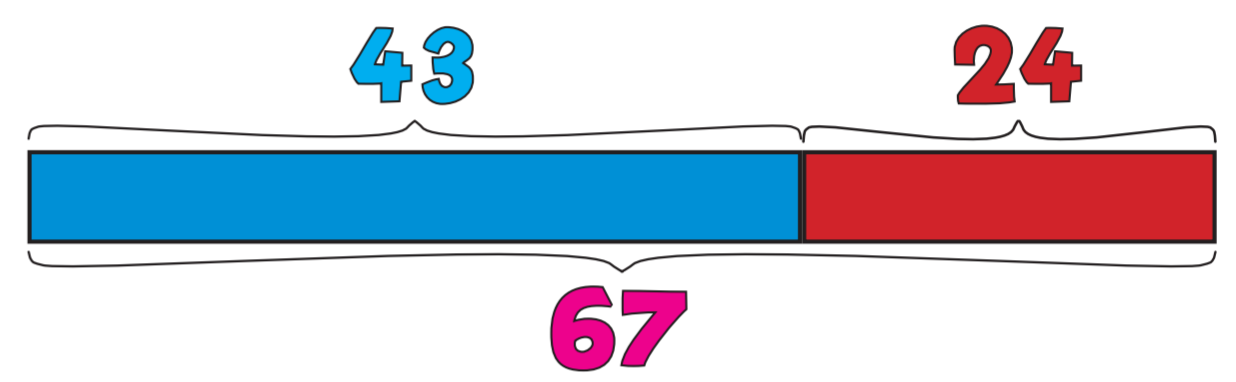
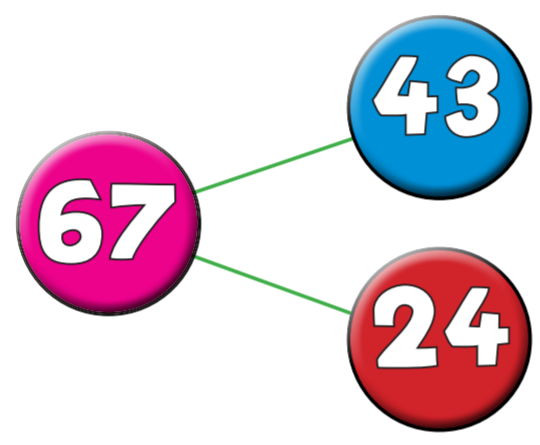
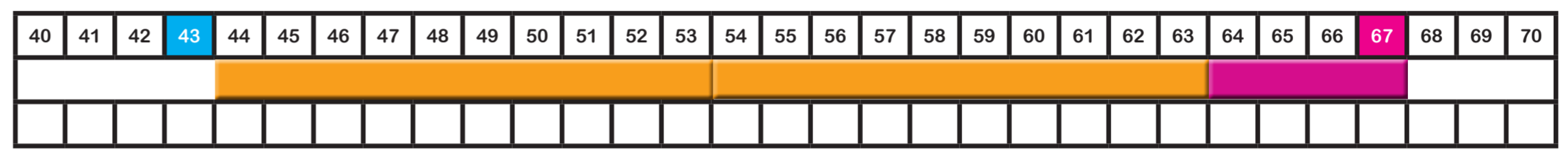
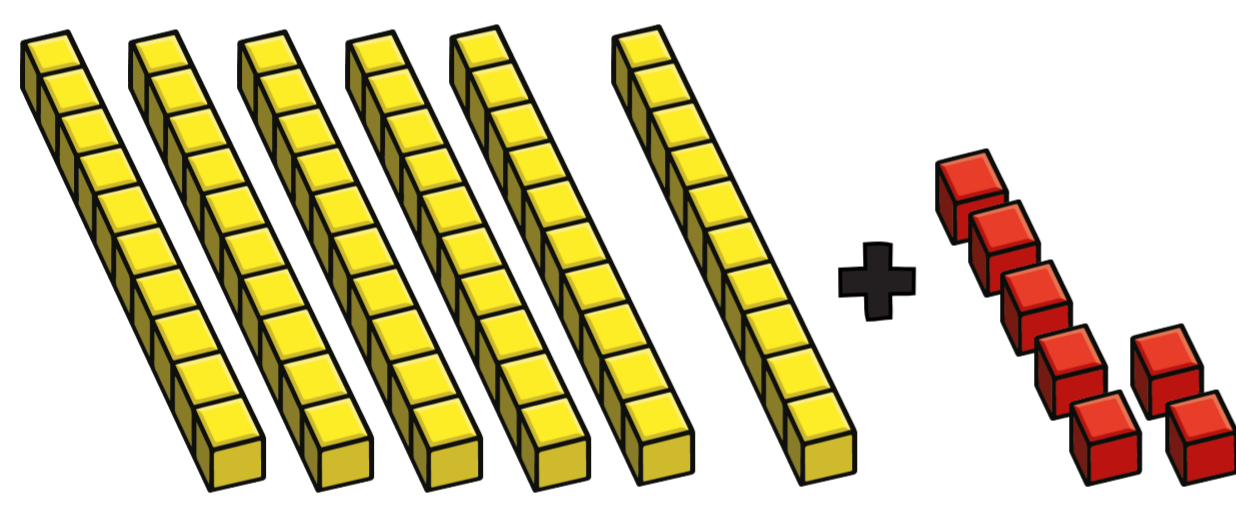
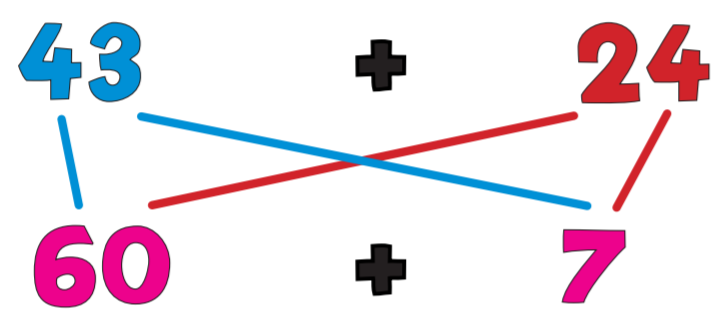
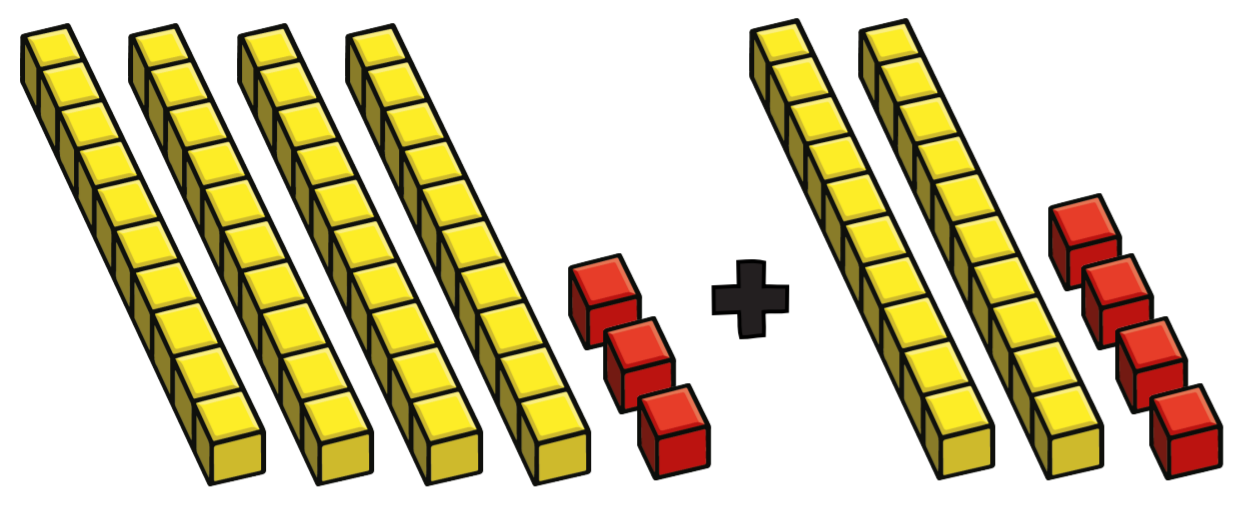


41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	

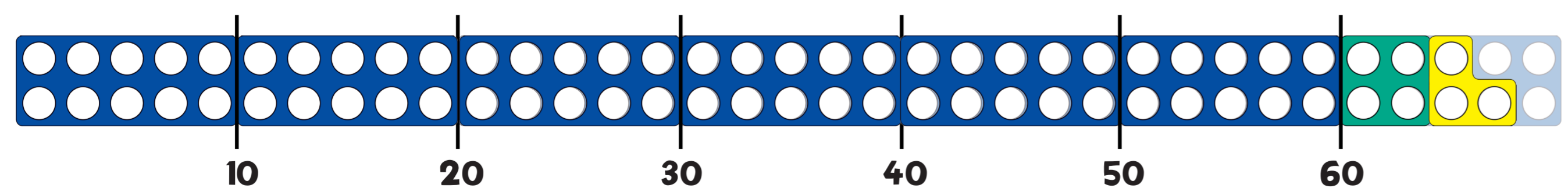
$$63 = 57 + 6$$



43 + 24 = 67



67 = 43 + 24



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80



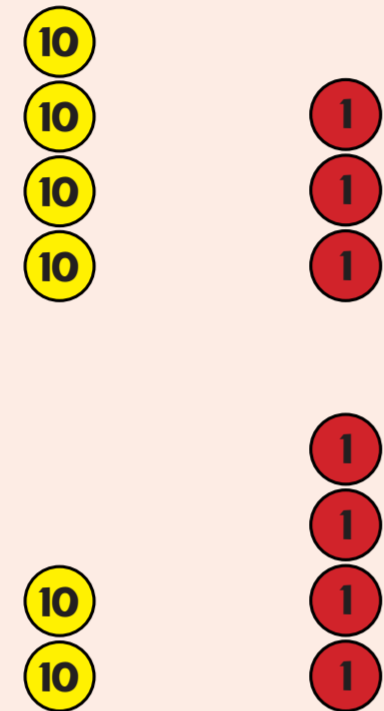
$$43 + 24 = 67$$



Stage 1

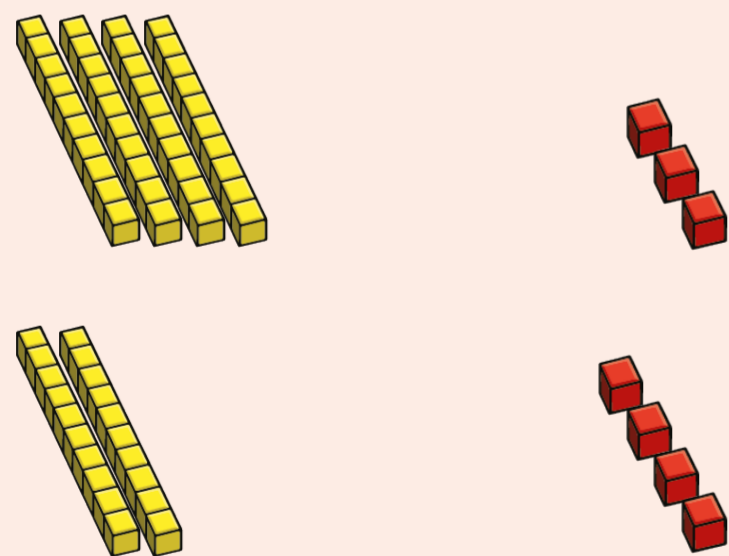
$$\begin{array}{r} 10 \quad 1 \\ 43 \\ + 24 \\ \hline \end{array}$$

Tens Ones



Tens

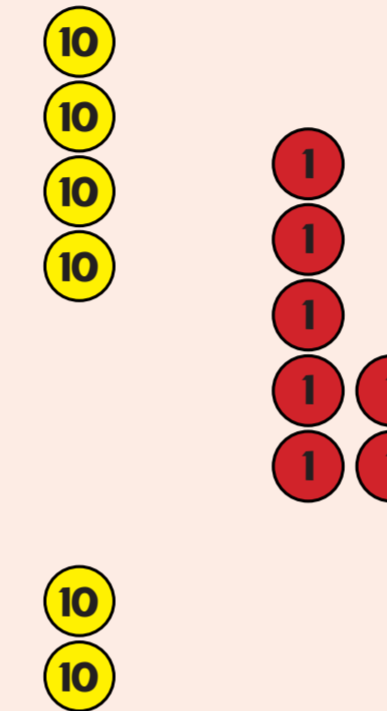
Ones



Stage 2

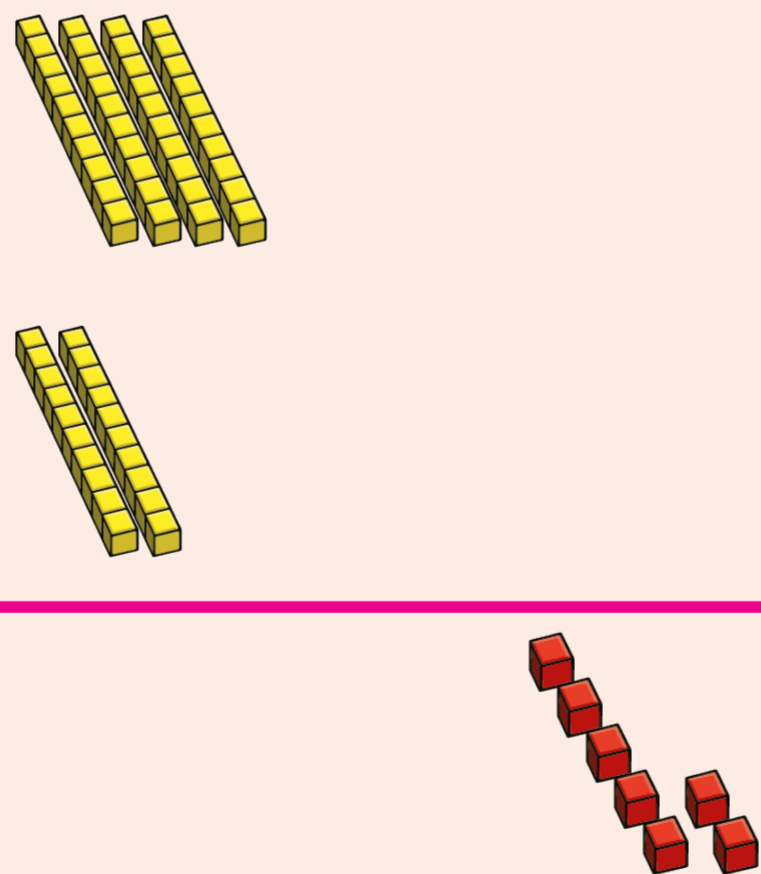
$$\begin{array}{r} 10 \quad 1 \\ 43 \\ + 24 \\ \hline 7 \end{array}$$

Tens Ones



Tens

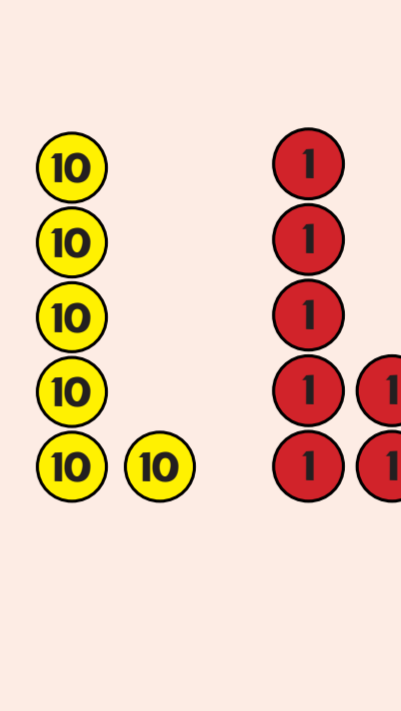
Ones



Stage 3

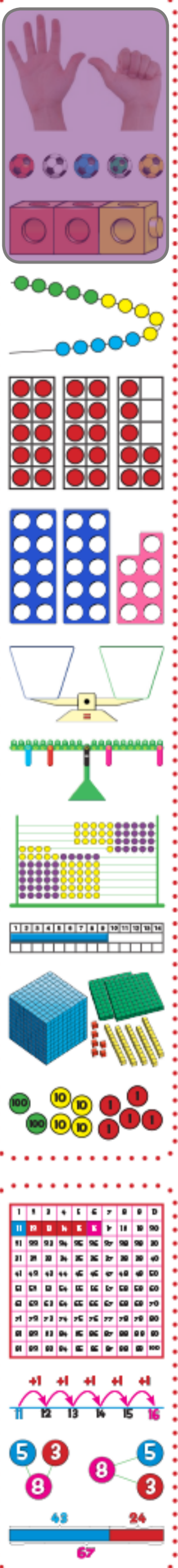
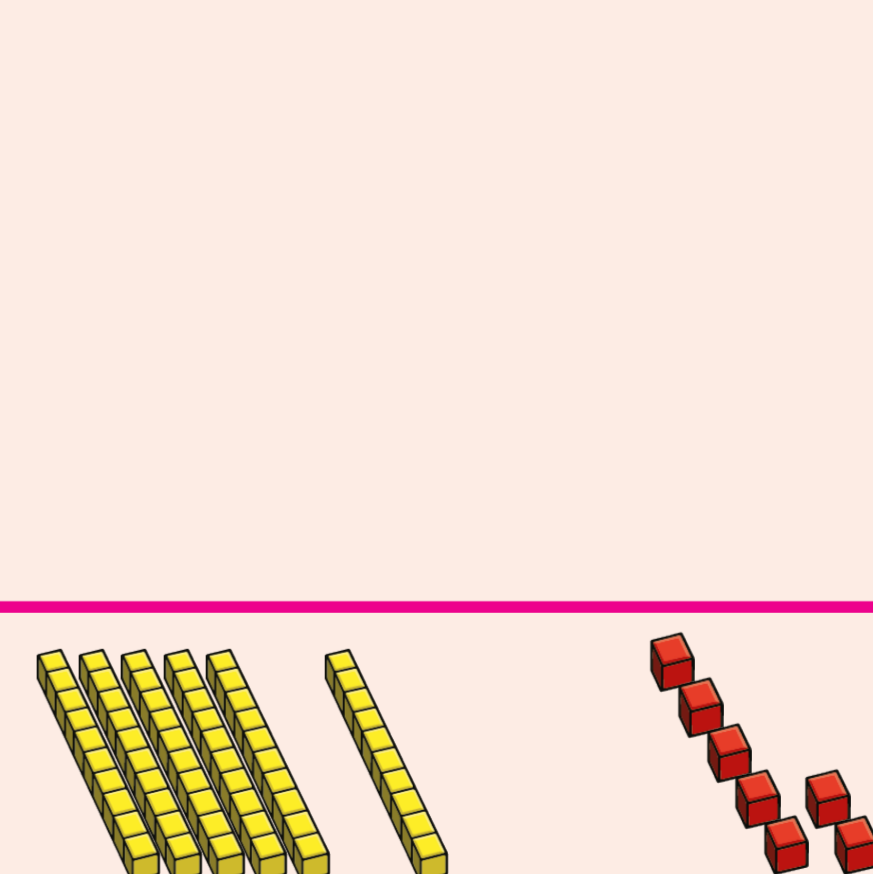
$$\begin{array}{r} 10 \quad 1 \\ 43 \\ + 24 \\ \hline 67 \end{array}$$

Tens Ones

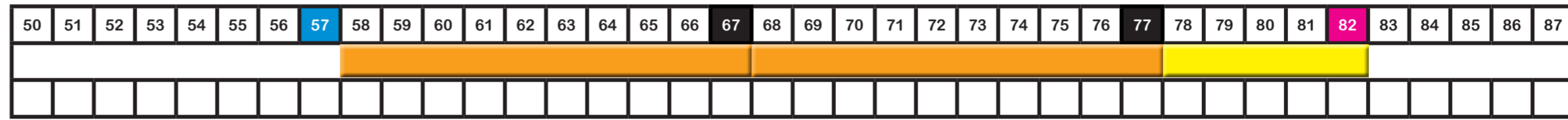
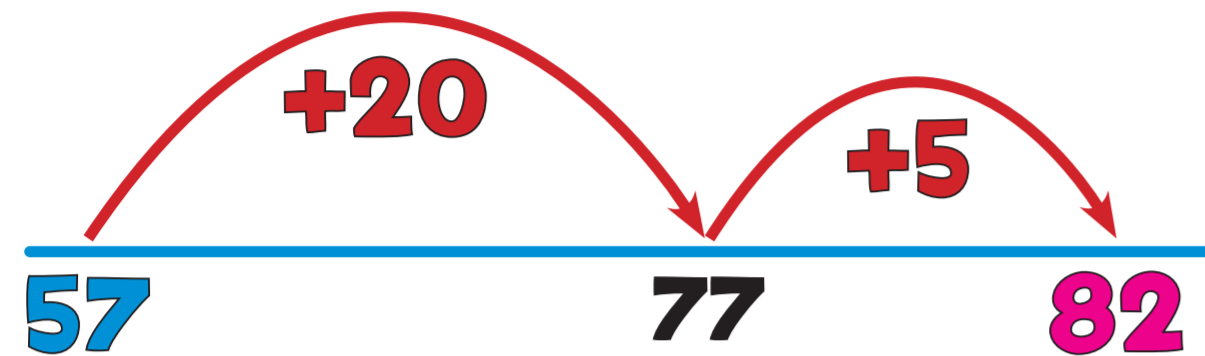


Tens

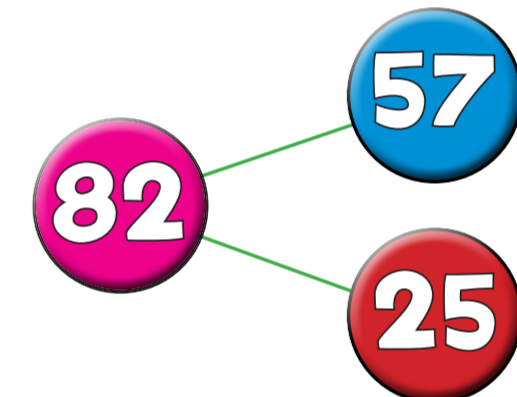
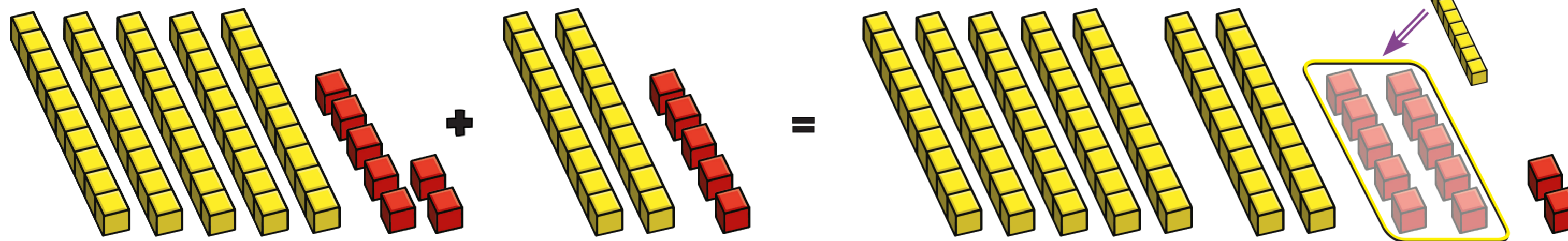
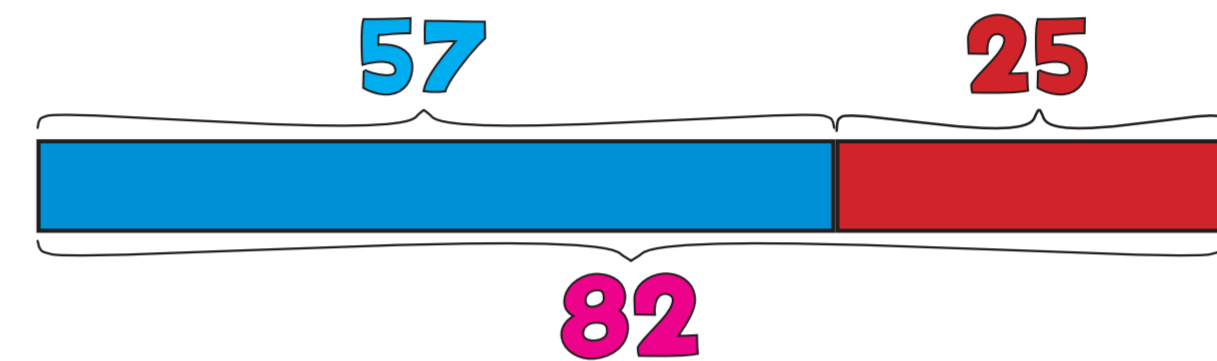
Ones



57 + 25 = 82



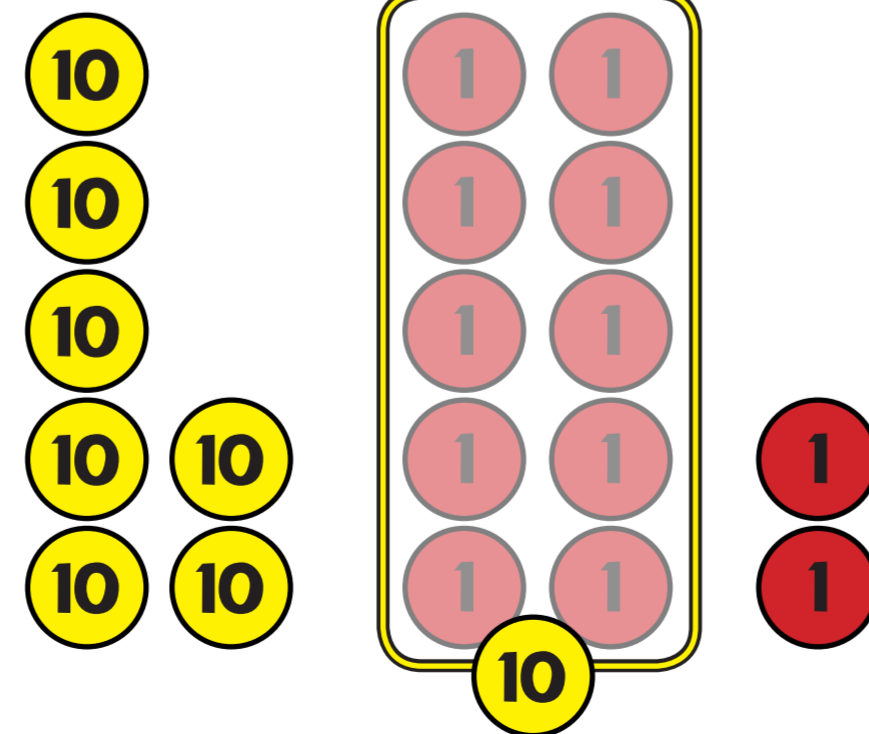
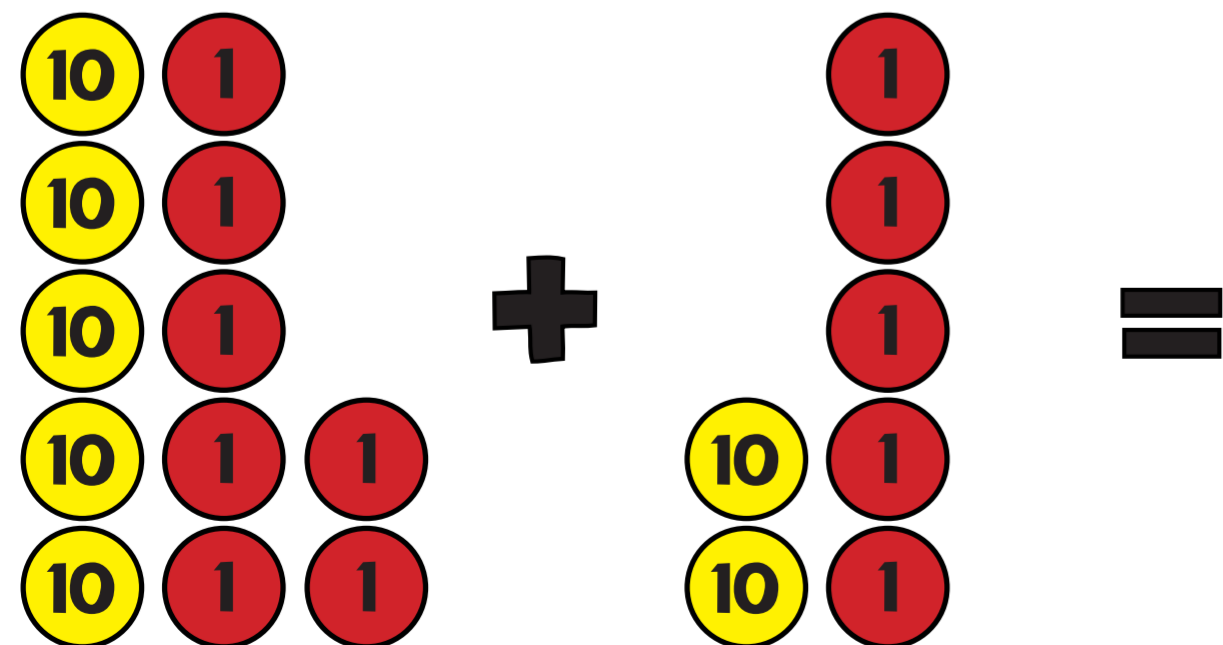
82 = 57 + 25



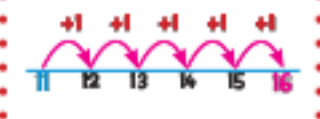
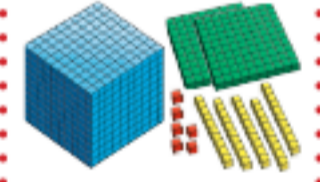
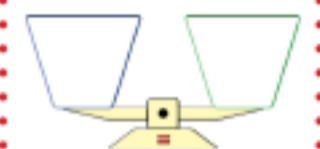
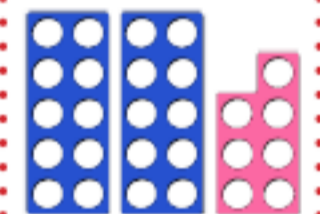
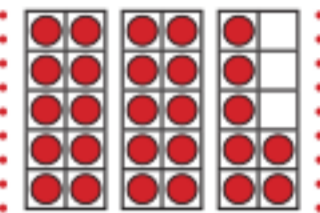
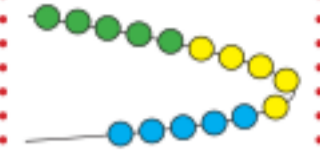
57

25

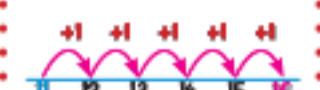
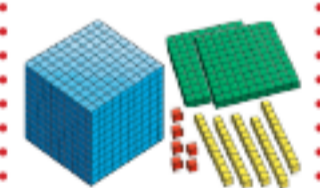
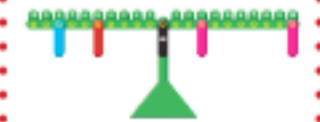
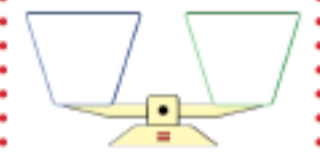
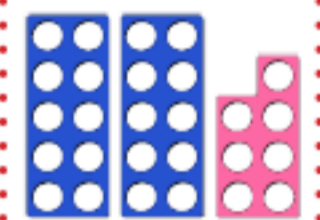
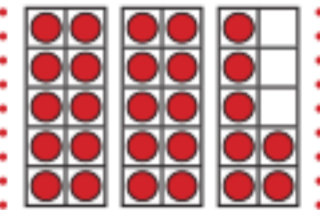
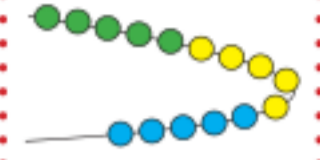
70 + 10 + 2



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



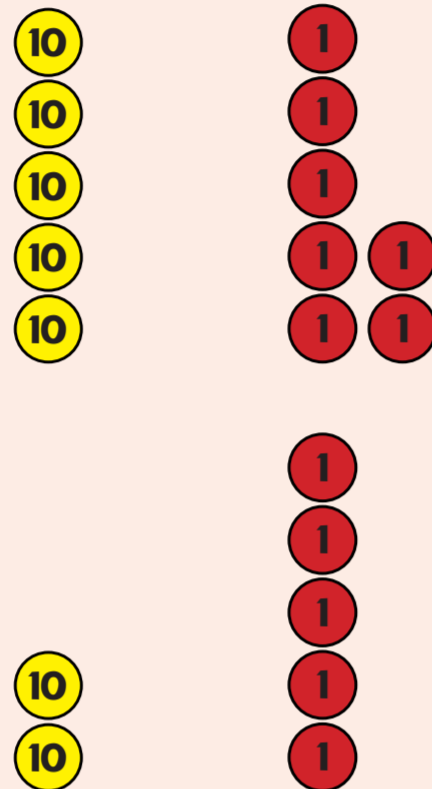
$$57 + 25 = 82$$



Stage 1

$$\begin{array}{r} 10 \quad 1 \\ 57 \\ + 25 \\ \hline \end{array}$$

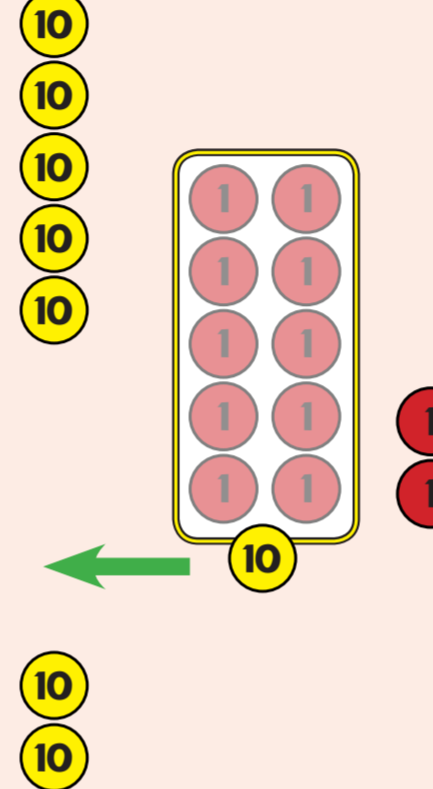
Tens Ones



Stage 2

$$\begin{array}{r} 10 \quad 1 \\ 57 \\ + 25 \\ \hline 2 \end{array}$$

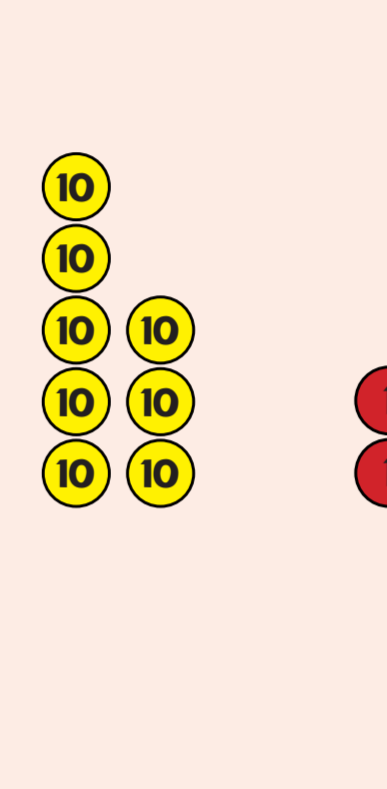
Tens Ones



Stage 3

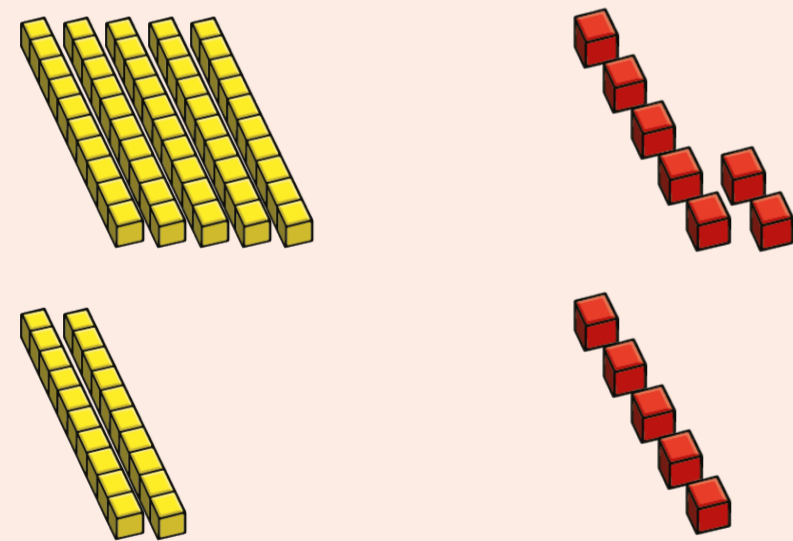
$$\begin{array}{r} 10 \quad 1 \\ 57 \\ + 25 \\ \hline 82 \end{array}$$

Tens Ones



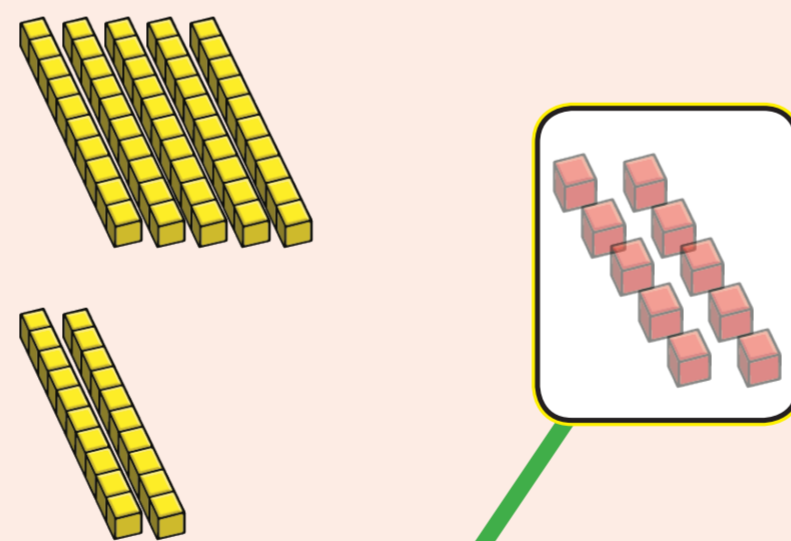
Tens

Ones



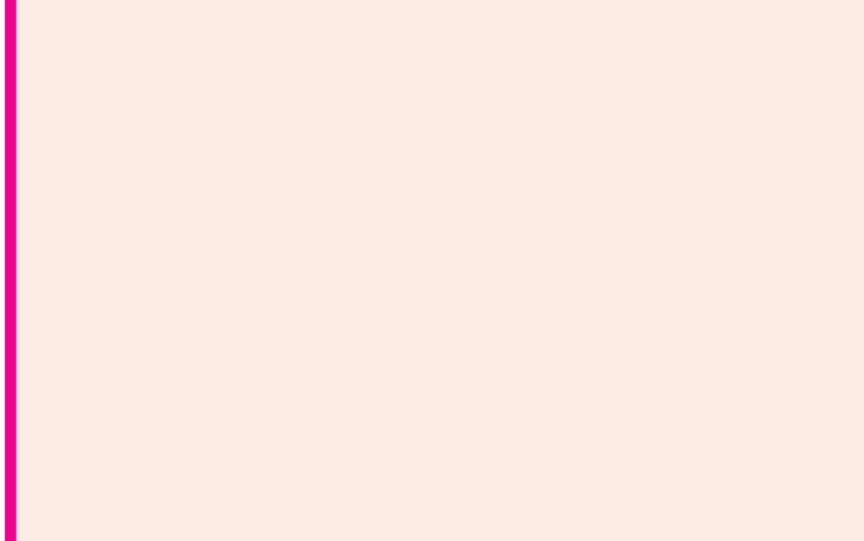
Tens

Ones

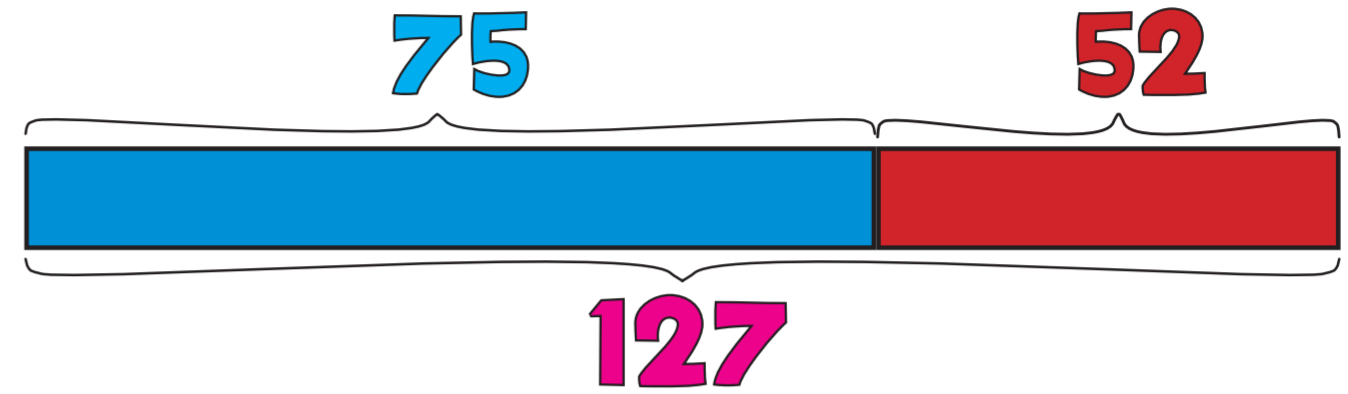
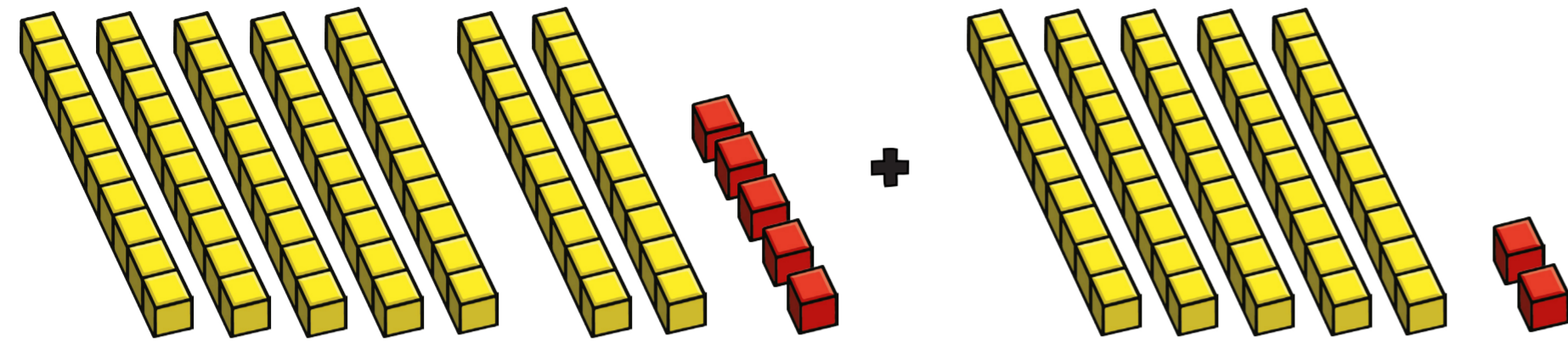


Tens

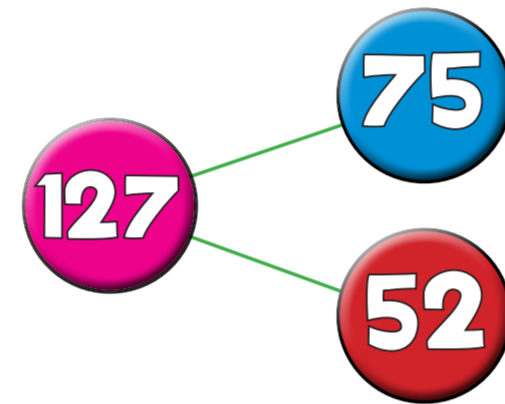
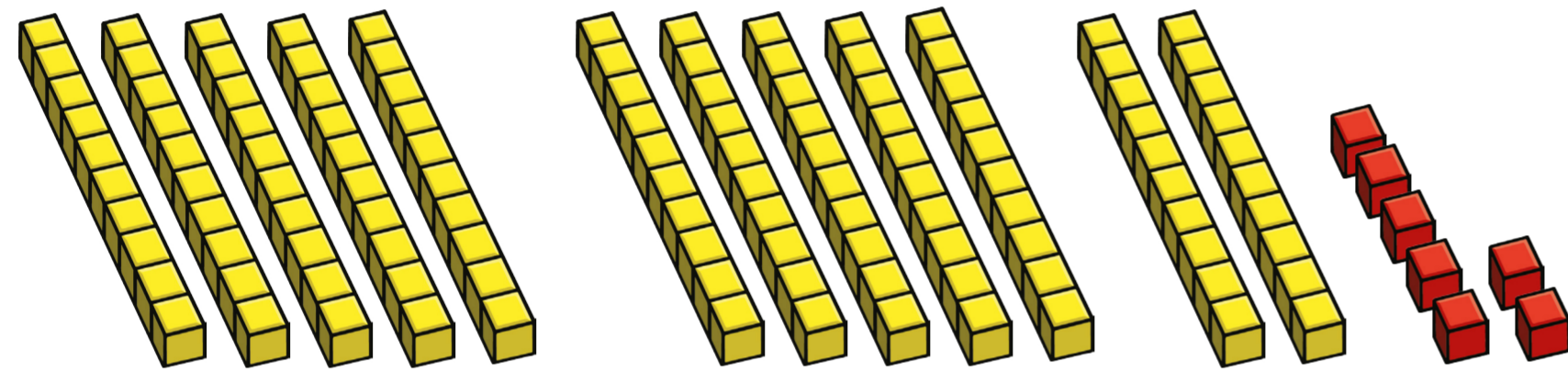
Ones



$$75 + 52 = 127$$

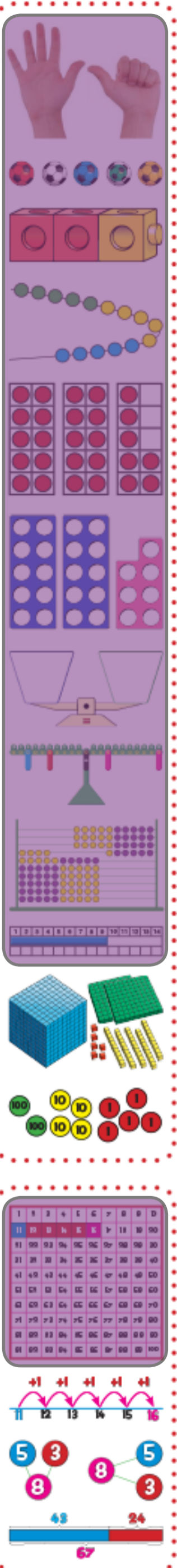
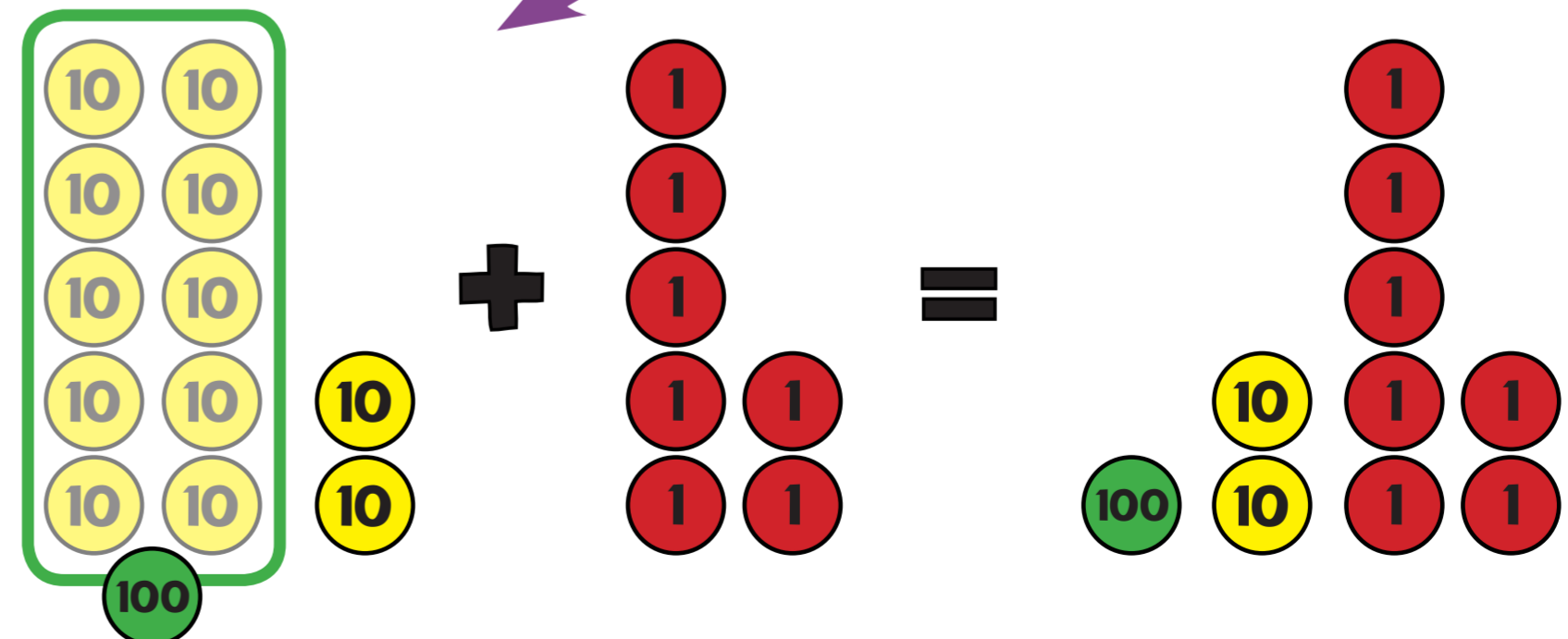
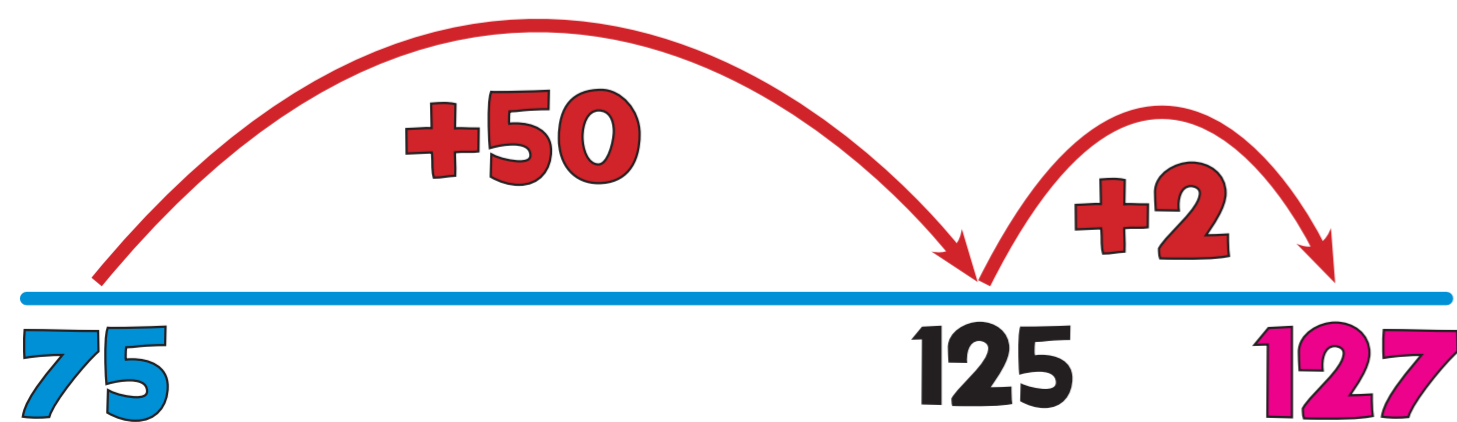
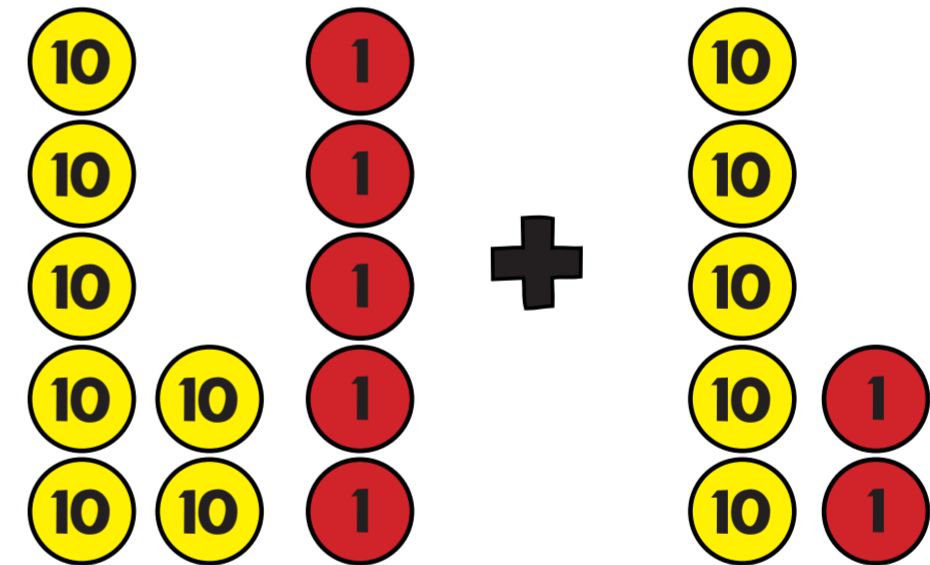
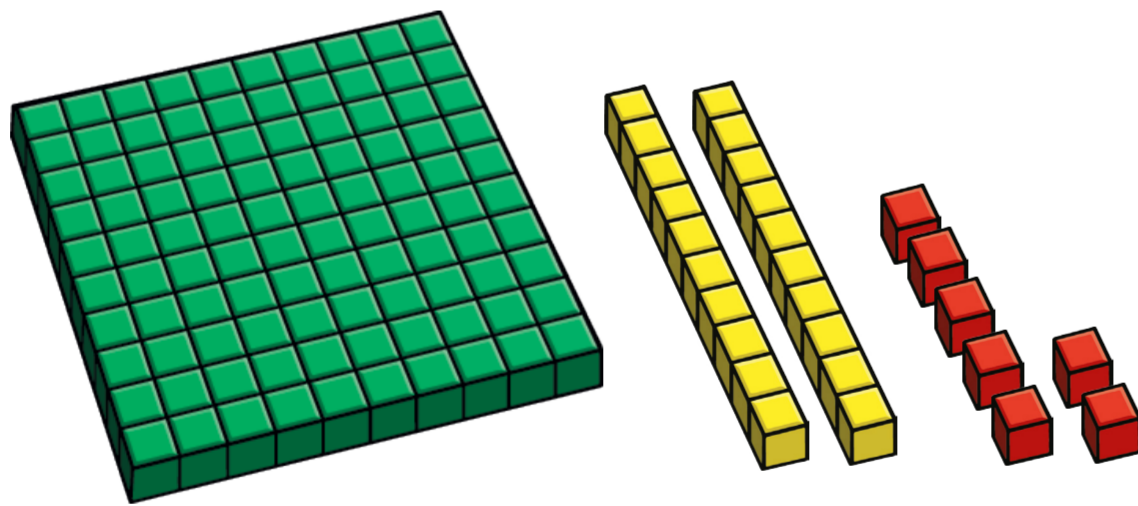


“Partition into 10s and 1s.”



$$127 = 75 + 52$$

“Exchange 10, 10s for a 100.”



$$75 + 52 = 127$$

1

Hundreds	Tens	Ones

$$\begin{array}{r} 100 \quad 10 \quad 1 \\ 75 \\ + 52 \\ \hline \end{array}$$

2

Hundreds	Tens	Ones

$$\begin{array}{r} 100 \quad 10 \quad 1 \\ 75 \\ + 52 \\ \hline 7 \end{array}$$

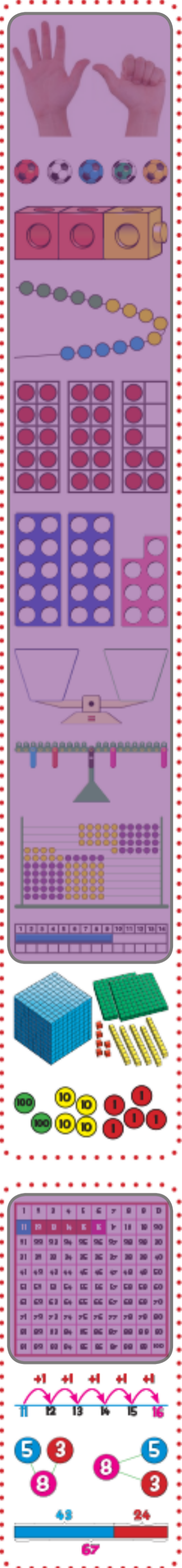
3

Hundreds	Tens	Ones

$$\begin{array}{r} 100 \quad 10 \quad 1 \\ 75 \\ + 52 \\ \hline 27 \\ 1 \end{array}$$

4

Hundreds	Tens	Ones

$$\begin{array}{r} 100 \quad 10 \quad 1 \\ 75 \\ + 52 \\ \hline 127 \\ 1 \end{array}$$


$$75 + 52 = 127$$

1

Hundreds	Tens	Ones
100	10	1
75	52	
+		

3

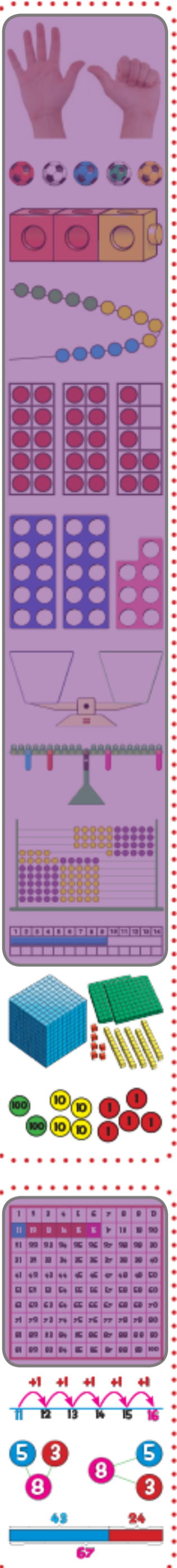
Hundreds	Tens	Ones
100	10	1
75	52	
+		

2

Hundreds	Tens	Ones
100	10	1
75	52	
+		
		7

4

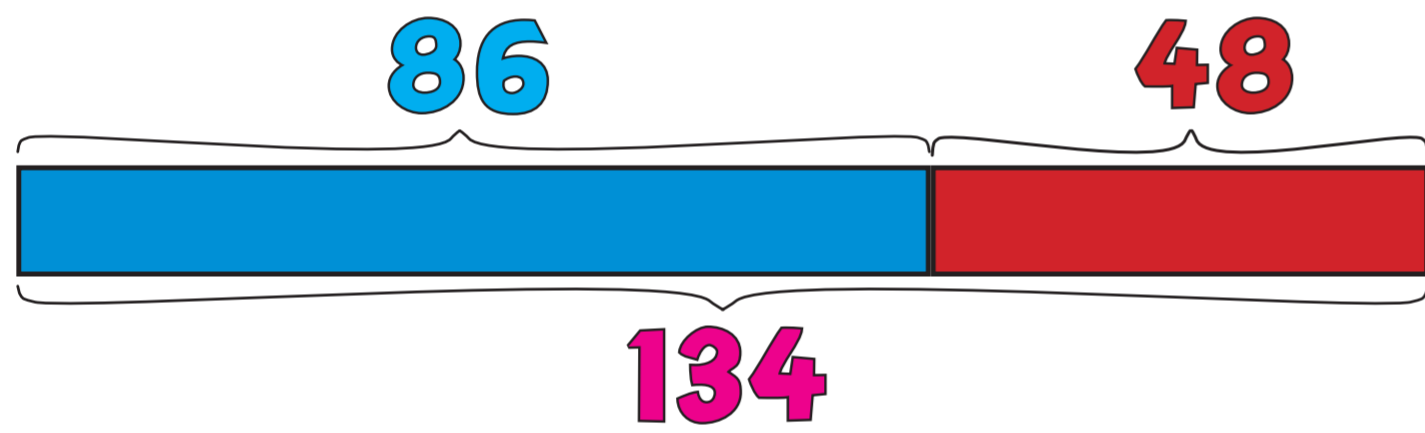
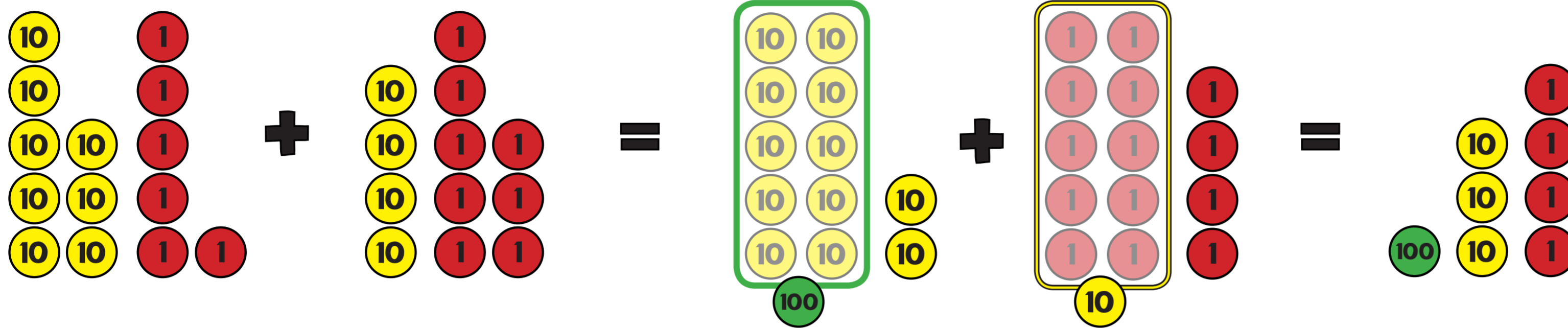
Hundreds	Tens	Ones
100	10	1
75	52	
+		
1	27	



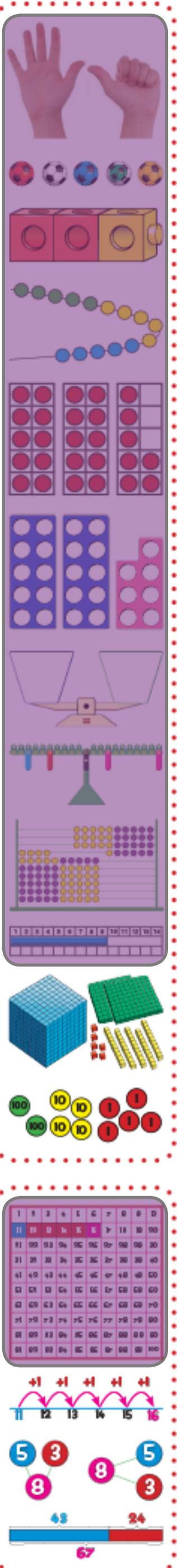
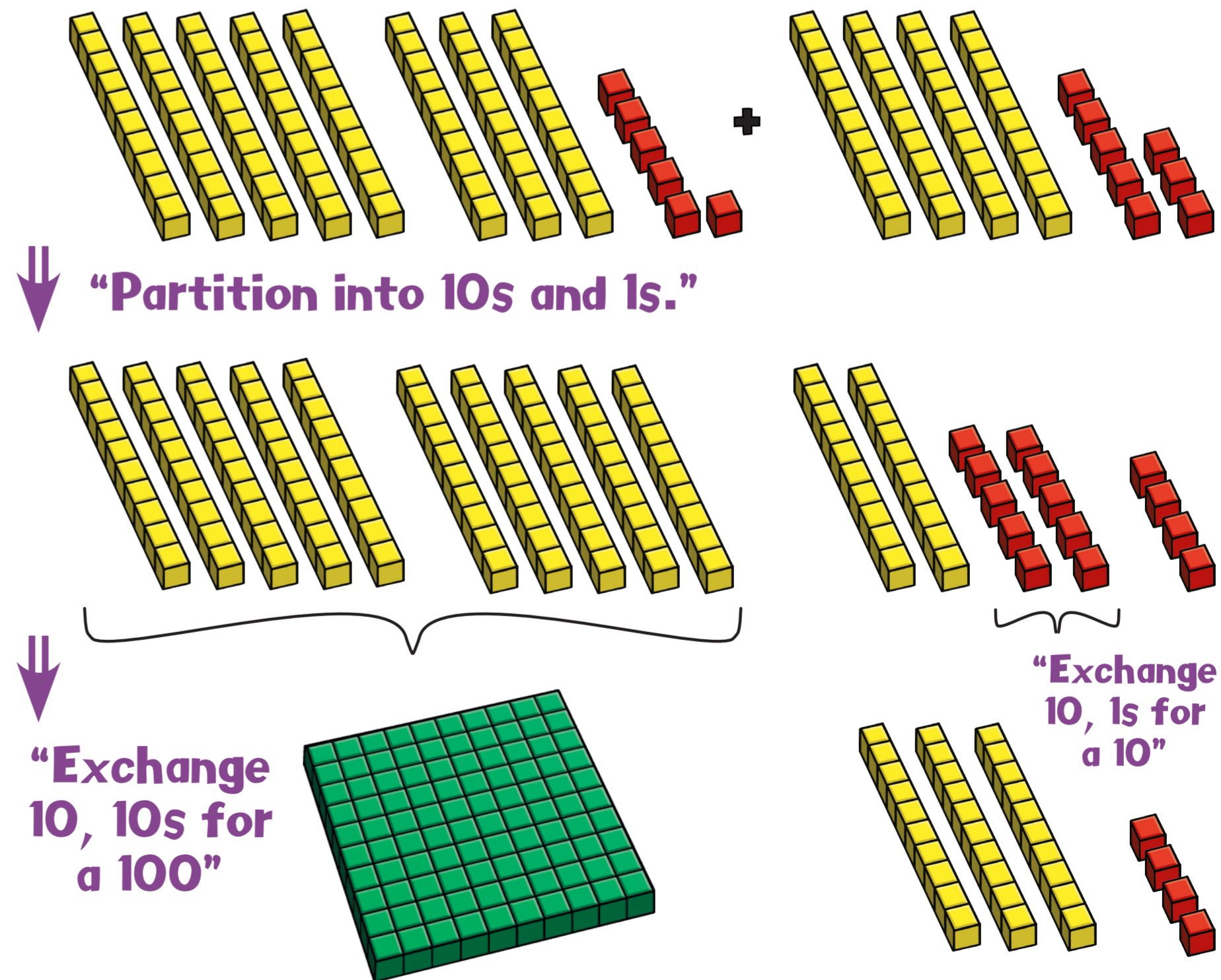
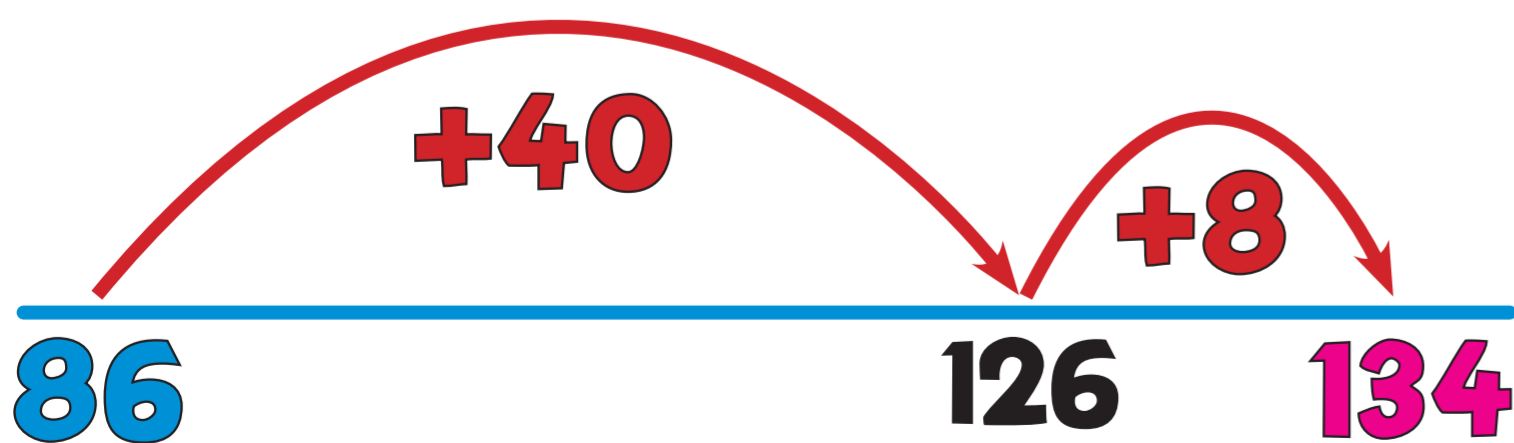
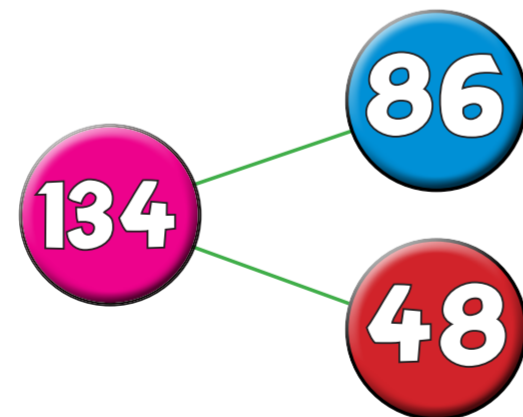
CPA
reasoning

$$86 + 48 = 134$$

Part 1



$$134 = 86 + 48$$



$$86 + 48 = 134$$

1

	Hundreds	Tens	Ones
100	8	6	
10		4	8
1			

2

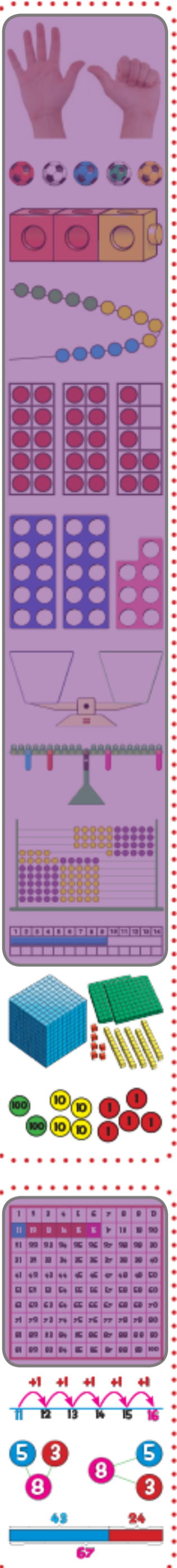
	Hundreds	Tens	Ones
100	8	6	
10		4	8
1			4

3

	Hundreds	Tens	Ones
100	8	6	
10		4	8
1		3	4

4

	Hundreds	Tens	Ones
100	1	3	4
10		3	4
1			



$$86 + 48 = 134$$

1

Hundreds	Tens	Ones
100	10 10 10 10	1 1 1 1 1 1
100	10 10 10 10	1 1 1 1

2

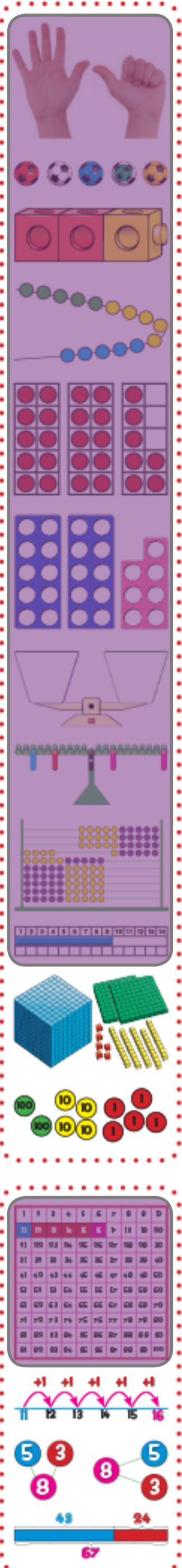
Hundreds	Tens	Ones
100	10 10 10 10	1 1 1 1 1 1
100	10 10 10 10	1 1 1 1

3

Hundreds	Tens	Ones
100	10 10 10 10 10 10	1 1 1 1
100	10 10 10 10	1 1 1 1

4

Hundreds	Tens	Ones
100	10 10 10	1 1 1 1
100	10 10 10	1 1 1 1



$$687 + 248 = 935$$

1

	Hundreds	Tens	Ones
687	6	8	7
+ 248	2	4	8
<hr/>			

3

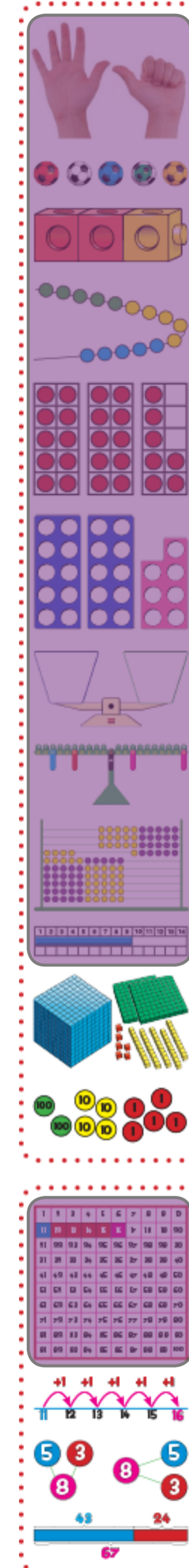
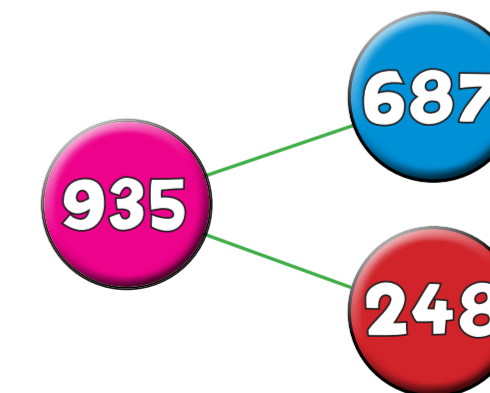
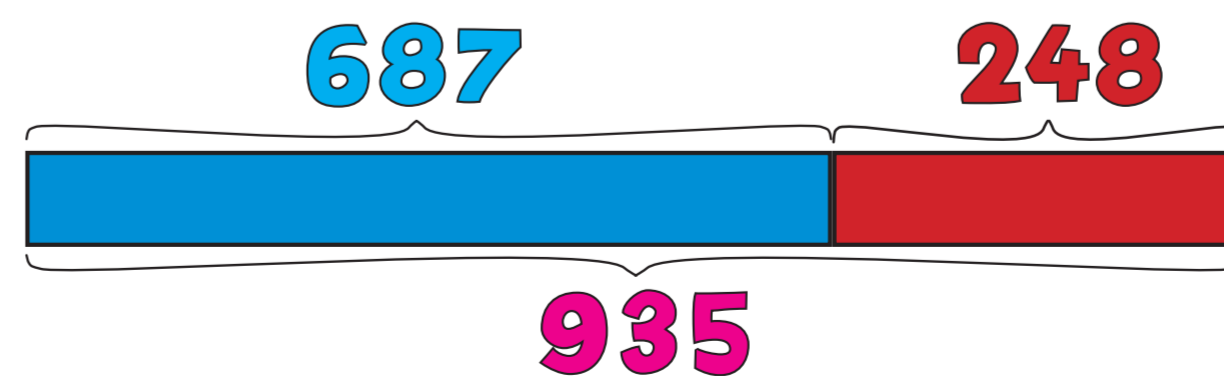
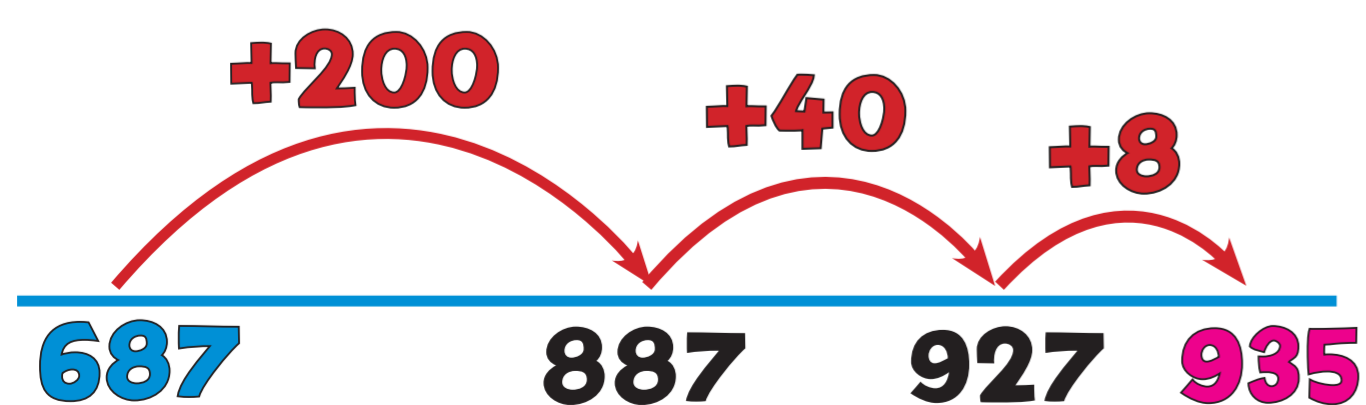
	Hundreds	Tens	Ones
687	6	8	7
+ 248	2	4	8
<hr/>			
		3	5
	1	1	

2

	Hundreds	Tens	Ones
687	6	8	7
+ 248	2	4	8
<hr/>			
			5
	1		

4

	Hundreds	Tens	Ones
687	6	8	7
+ 248	2	4	8
<hr/>			
	9	3	5
	1	1	



738 + 524 = 1262

1

1000	100	10	1
7	3	8	
+	5	2	4
<hr/>			
<hr/>			

Thousands Hundreds Tens Ones

2

1000	100	10	1
7	3	8	
+	5	2	4
<hr/>			
			2
<hr/>			
			1

Thousands Hundreds Tens Ones

3

1000	100	10	1
7	3	8	
+	5	2	4
<hr/>			
		6	2
<hr/>			
			1

Thousands Hundreds Tens Ones

4

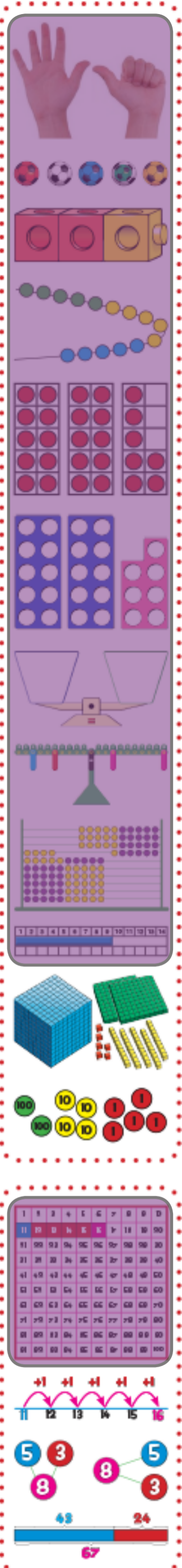
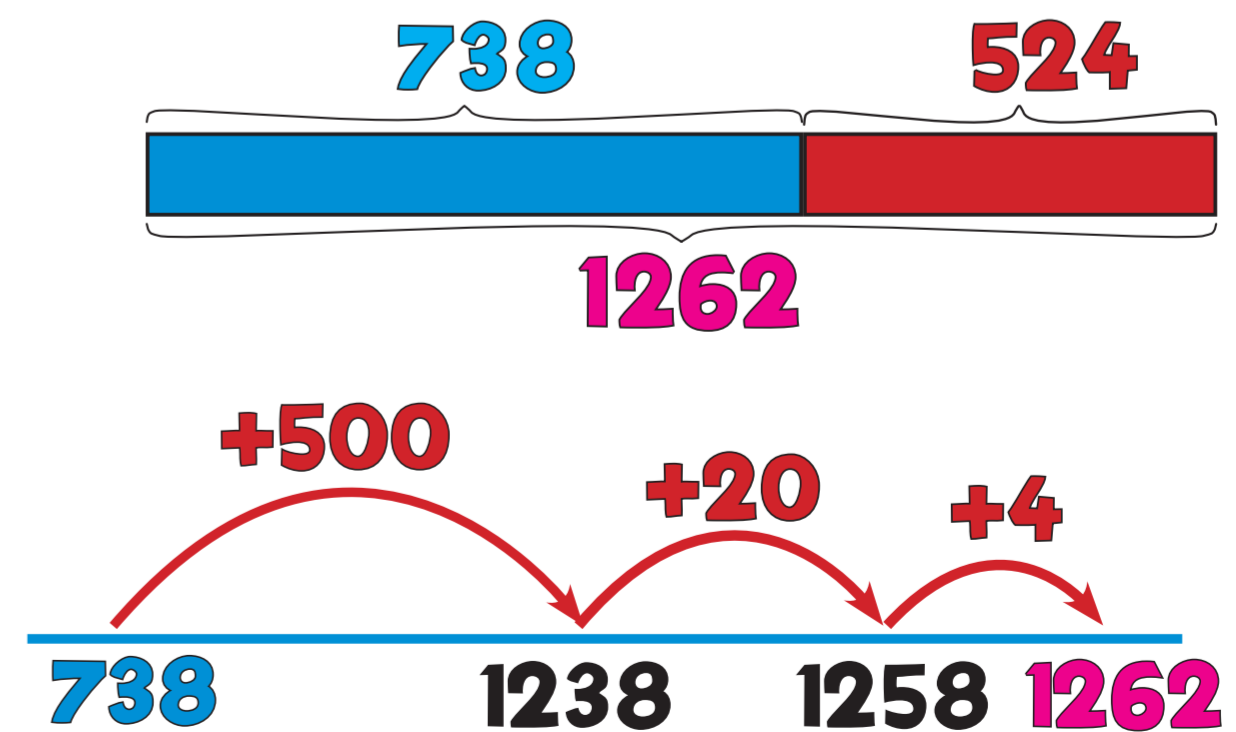
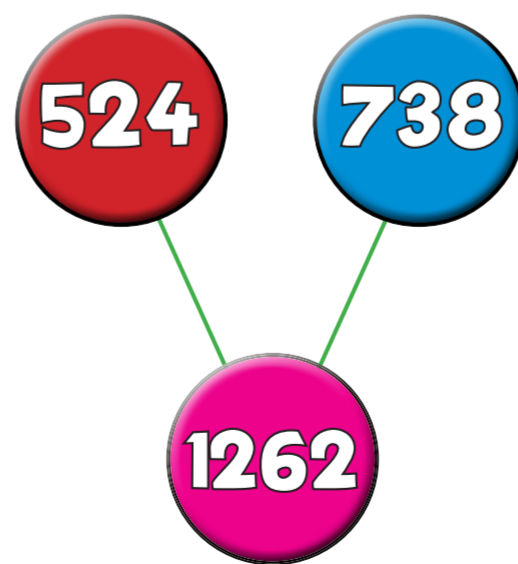
1000	100	10	1
7	3	8	
+	5	2	4
<hr/>			
2	6	2	
<hr/>			
1			1

Thousands Hundreds Tens Ones

5

1000	100	10	1
7	3	8	
+	5	2	4
<hr/>			
1	2	6	2
<hr/>			
1			1

Thousands Hundreds Tens Ones



$$4.8 + 3.8 = 8.6$$

1

Ones ■ Tenths

4.8
+ 3.8

3

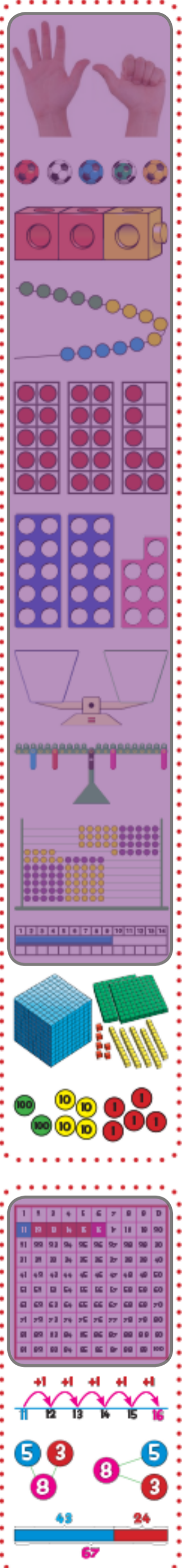
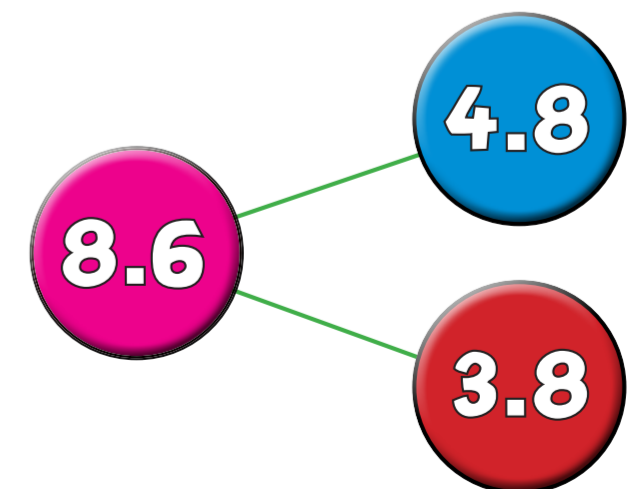
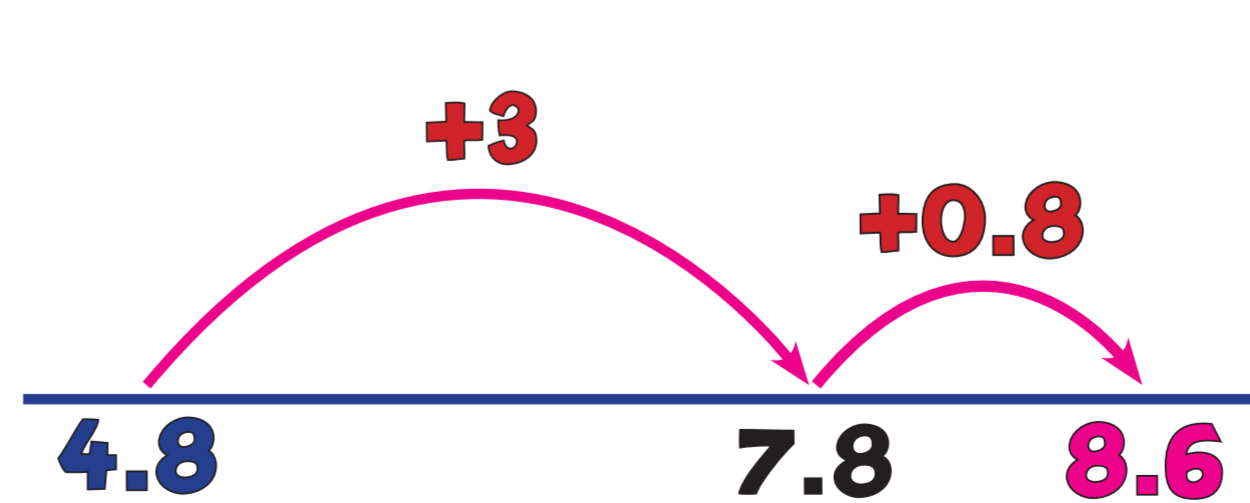
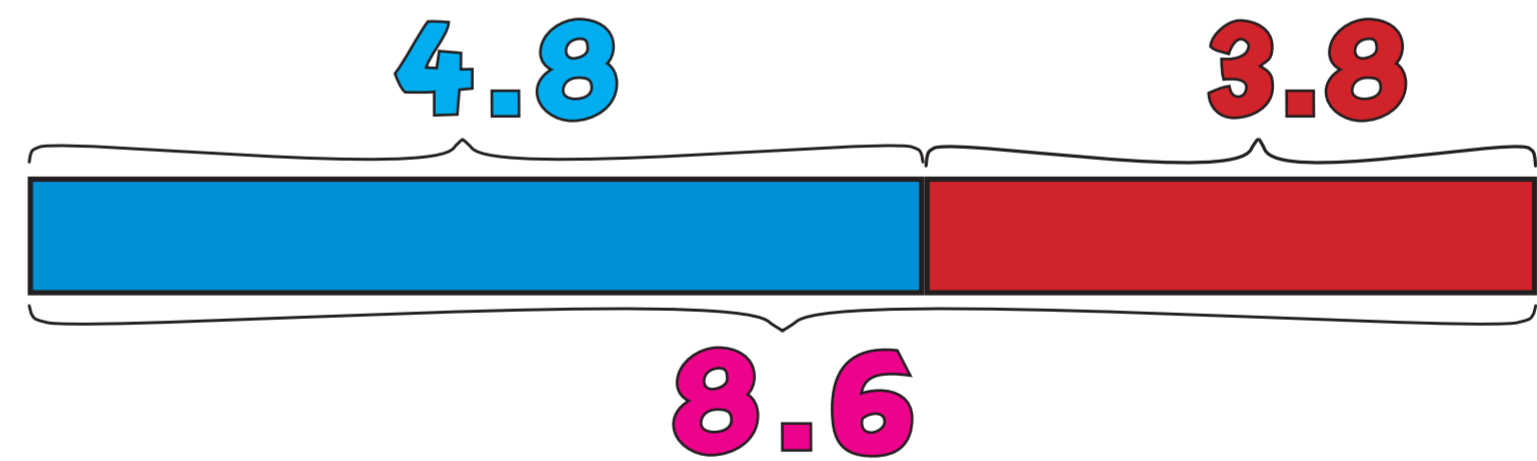
Ones ■ Tenths

4.8
+ 3.8
8.6

2

Ones ■ Tenths

4.8
+ 3.8
8.6



$$5.65 + 3.29 = 8.94$$

1

Ones	Tenths	Hundredths
1	0.1	0.01
1	0.1	0.01
1	0.1	0.01
1	0.1	0.01
1	0.1 0.1	0.01
		0.01
		0.01 0.01
1	0.1	0.01 0.01
1	0.1	0.01 0.01

2

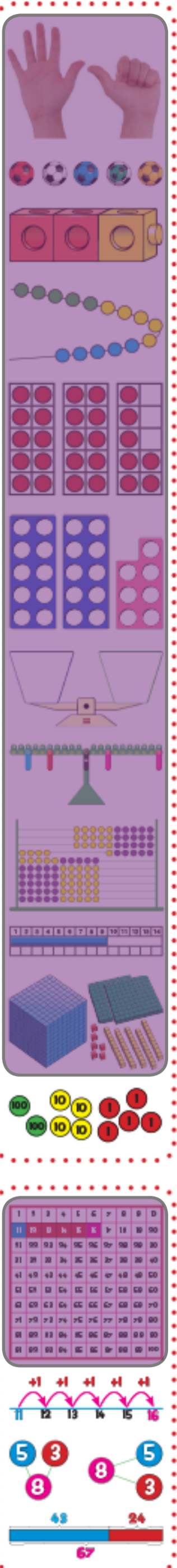
Ones	Tenths	Hundredths
1	0.1	0.01
1	0.1	0.01
1	0.1	0.01
1	0.1	0.01
1	0.1 0.1	0.01
		0.01 0.01
		0.01 0.01
		0.01 0.01
		0.01 0.01
		0.01 0.01
1	0.1	0.01
1	0.1	0.01

3

Ones	Tenths	Hundredths
1		
1		
1		
1		
1	0.1	
	0.1 0.1	0.01
	0.1 0.1	0.01
	0.1 0.1	0.01
	0.1 0.1	0.01
	0.1 0.1	0.01
1		
1		
1		

4

Ones	Tenths	Hundredths
1		
1		
1		
1		
1	0.1	
	0.1 0.1	0.01
	0.1 0.1	0.01
	0.1 0.1	0.01
	0.1 0.1	0.01
	0.1 0.1	0.01
1		
1		



$$76.7 + 58.5 = 135.2$$

1 100 10 1 $\frac{1}{10}$

76.7
+ 58.5

Hundreds Tens Ones Tenth

2 100 10 1 $\frac{1}{10}$

76.7
+ 58.5

Hundreds Tens Ones Tenth

3 100 10 1 $\frac{1}{10}$

76.7
+ 58.5

Hundreds Tens Ones Tenth

4 100 10 1 $\frac{1}{10}$

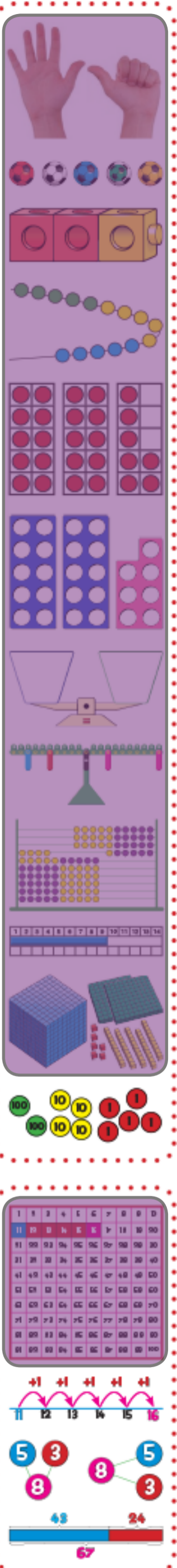
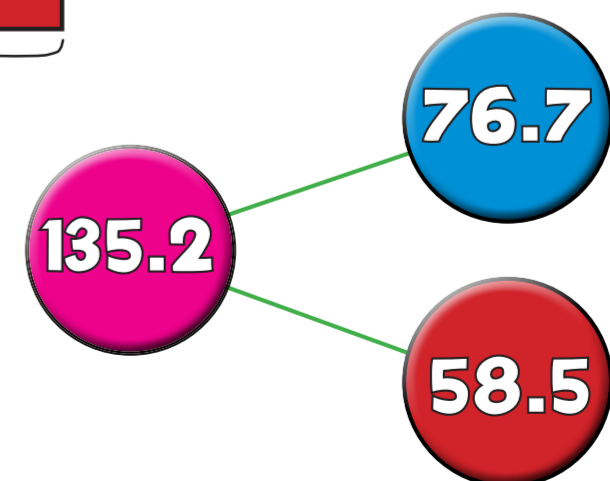
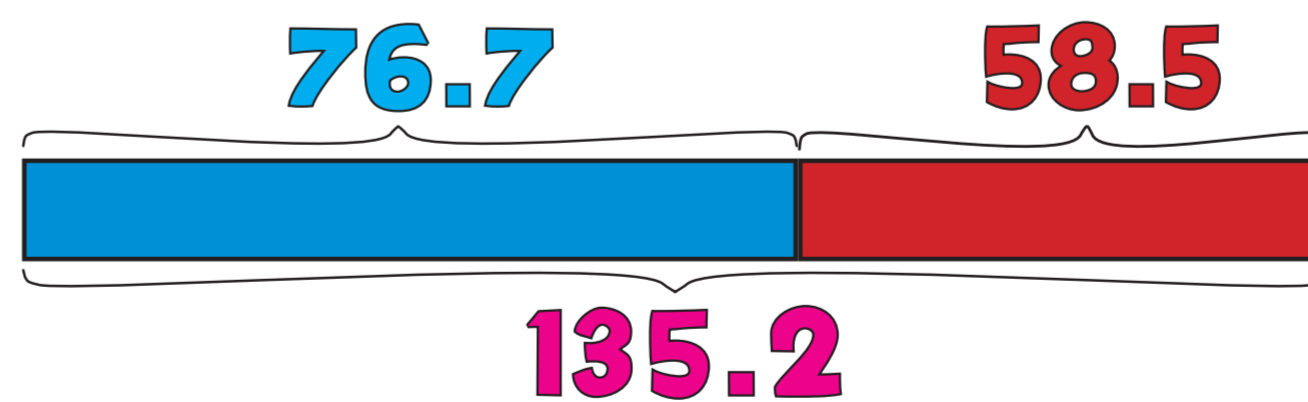
76.7
+ 58.5

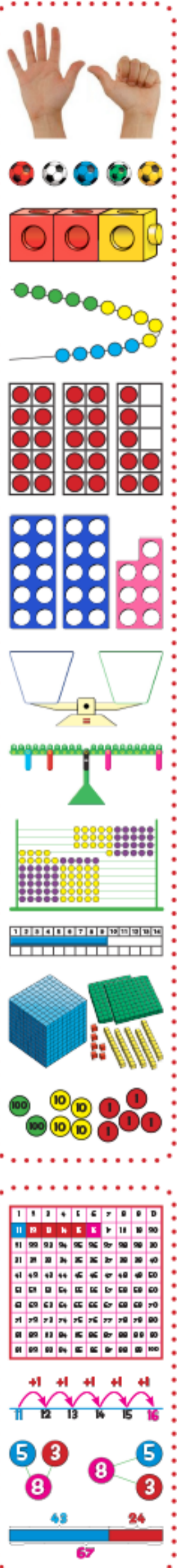
Hundreds Tens Ones Tenth

5 100 10 1 $\frac{1}{10}$

76.7
+ 58.5

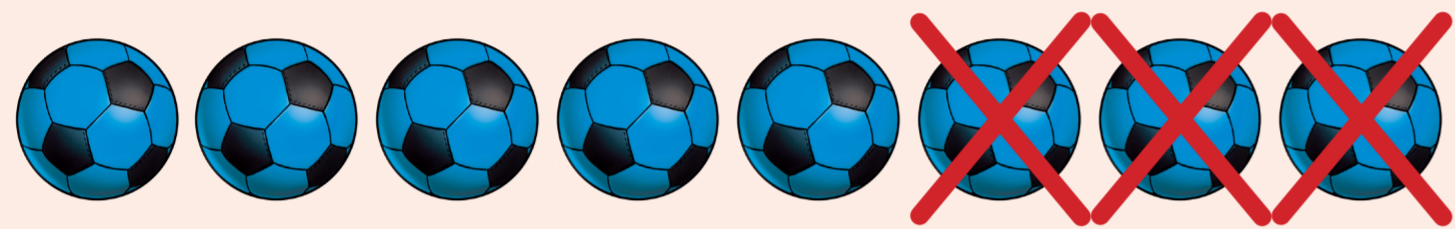
Hundreds Tens Ones Tenth





Removing Items

Take Away



"If I had 8 footballs and kicked 3 over the fence, how many did I have left?" "5"

Reduction

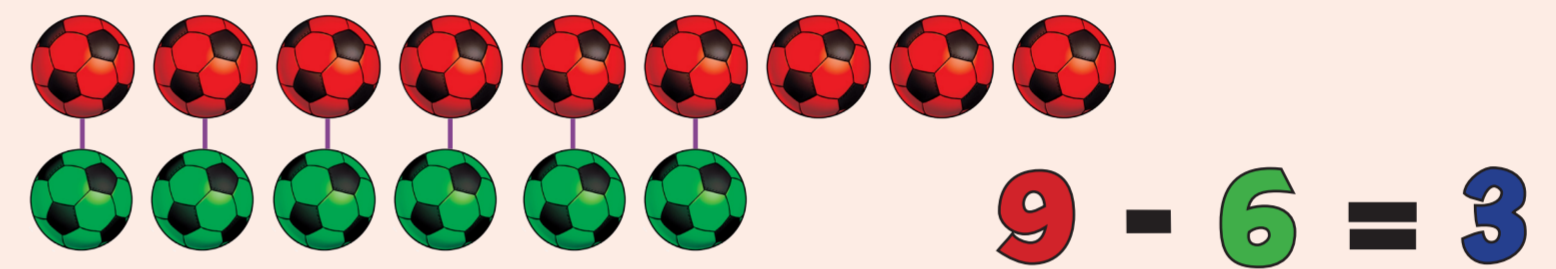
$$8 - 3 = 5$$



"The football cost £8 but I got £3 off in the sales. How much did I pay?" "£5"

Comparing Sets

Comparison



"If I had 9 footballs and you had 6, how many more balls have I got than you?" "3"

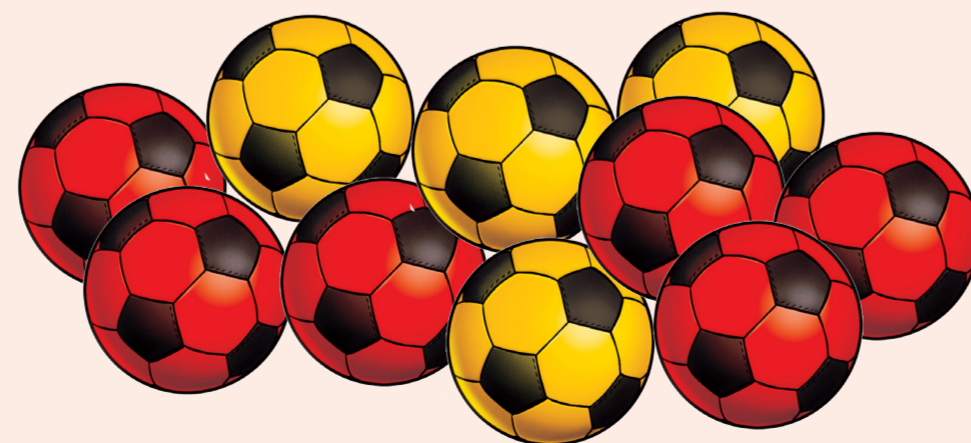
Inverse of Addition

"The football costs £9 but I've only got £6. How much more do I need?" "£3"



$$£6 + ? = £9$$

Whole / Part / Part



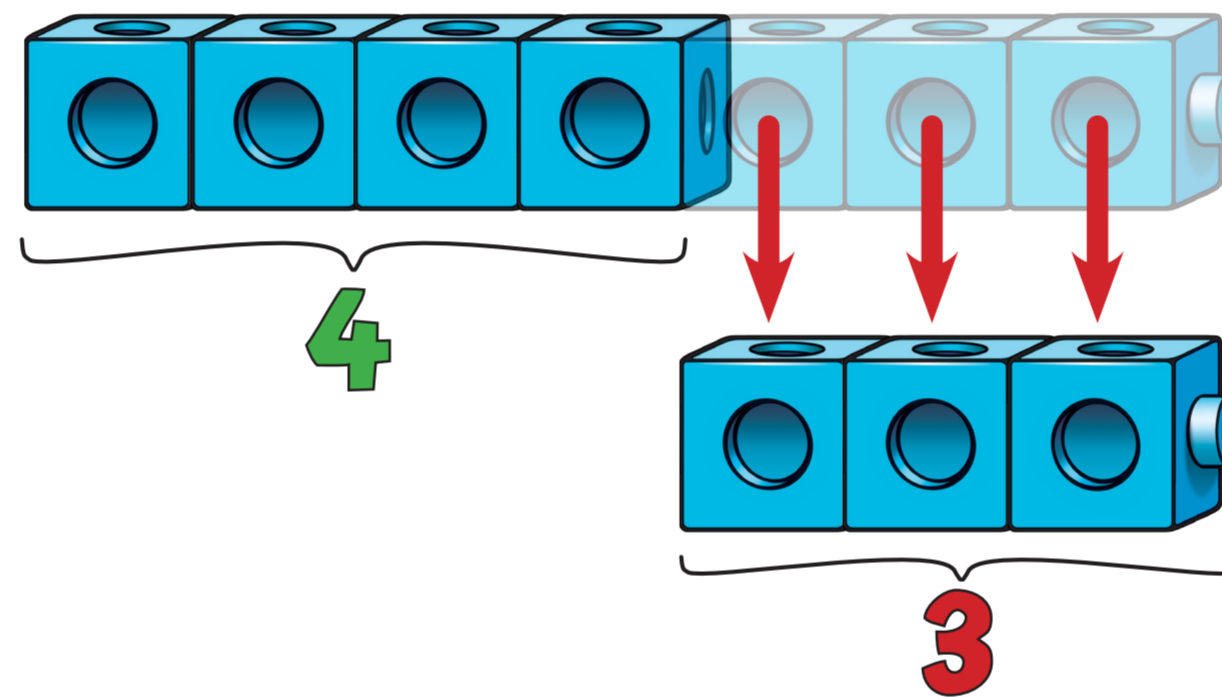
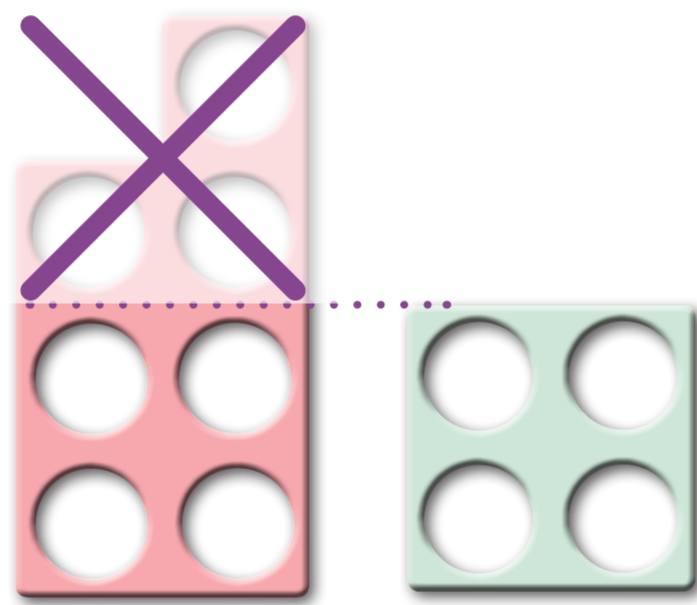
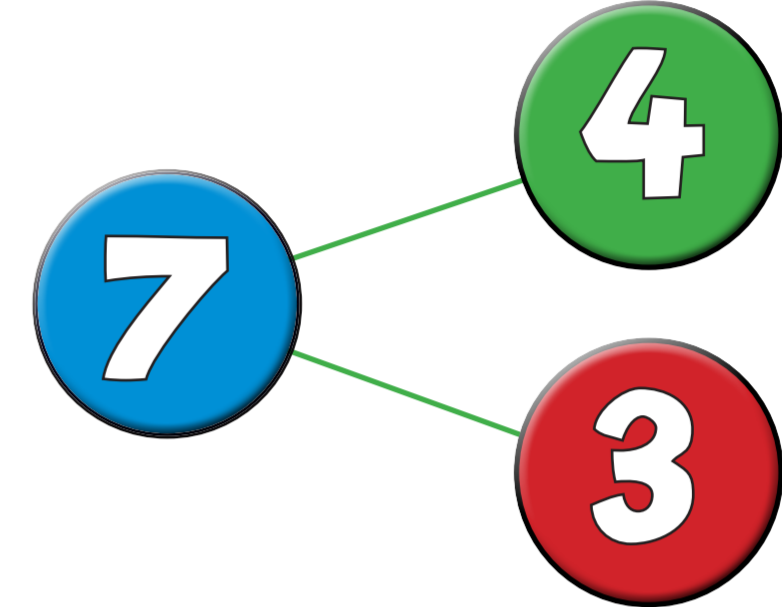
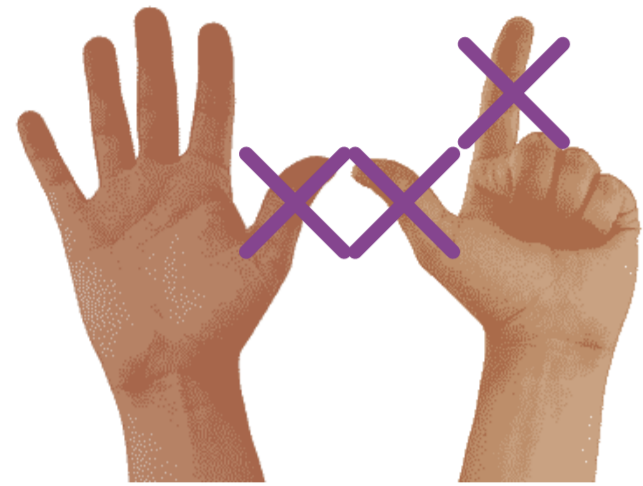
"There are 10 footballs in my bag. 6 are red, how many are yellow?" "4"

$$10 - 6 = 4$$

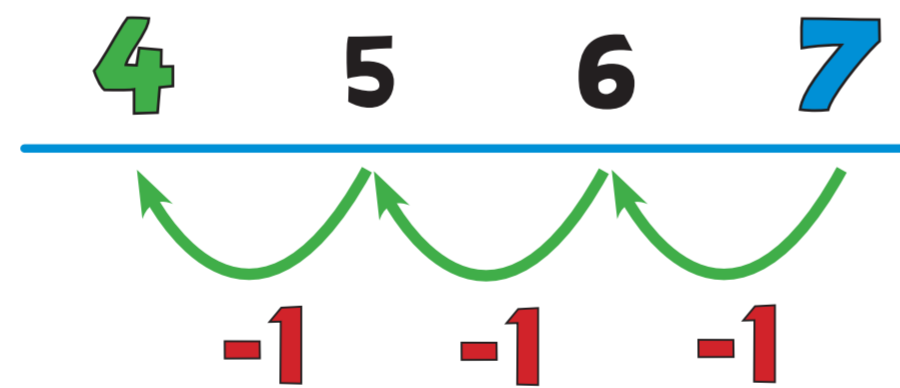
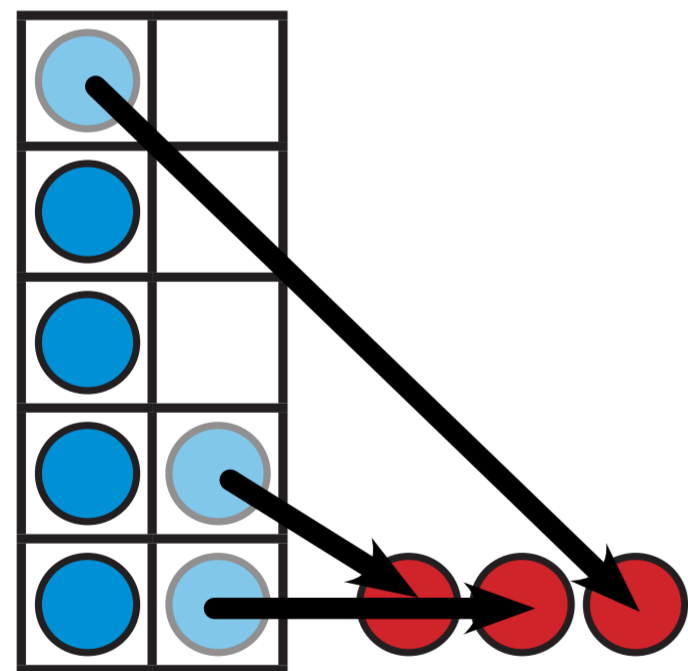
CPA
 ← reasoning →



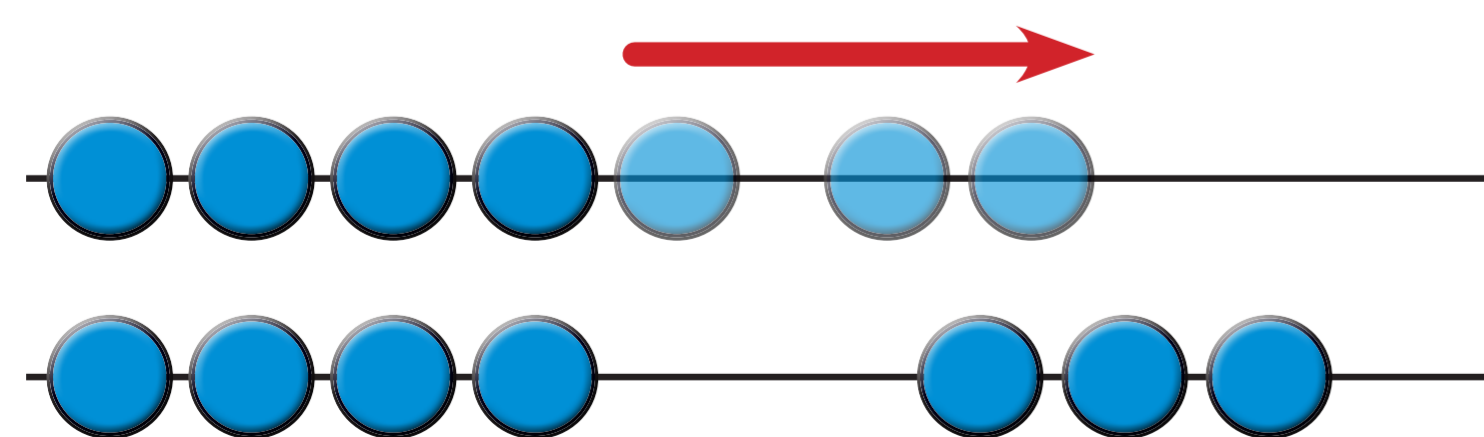
$$7 - 3 = 4$$



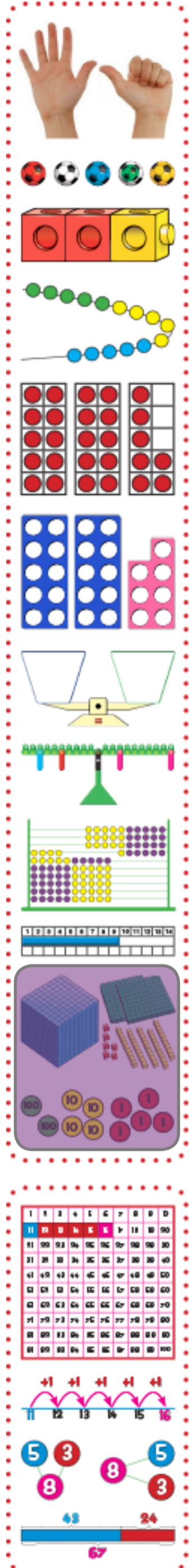
Count Back
 ← Images



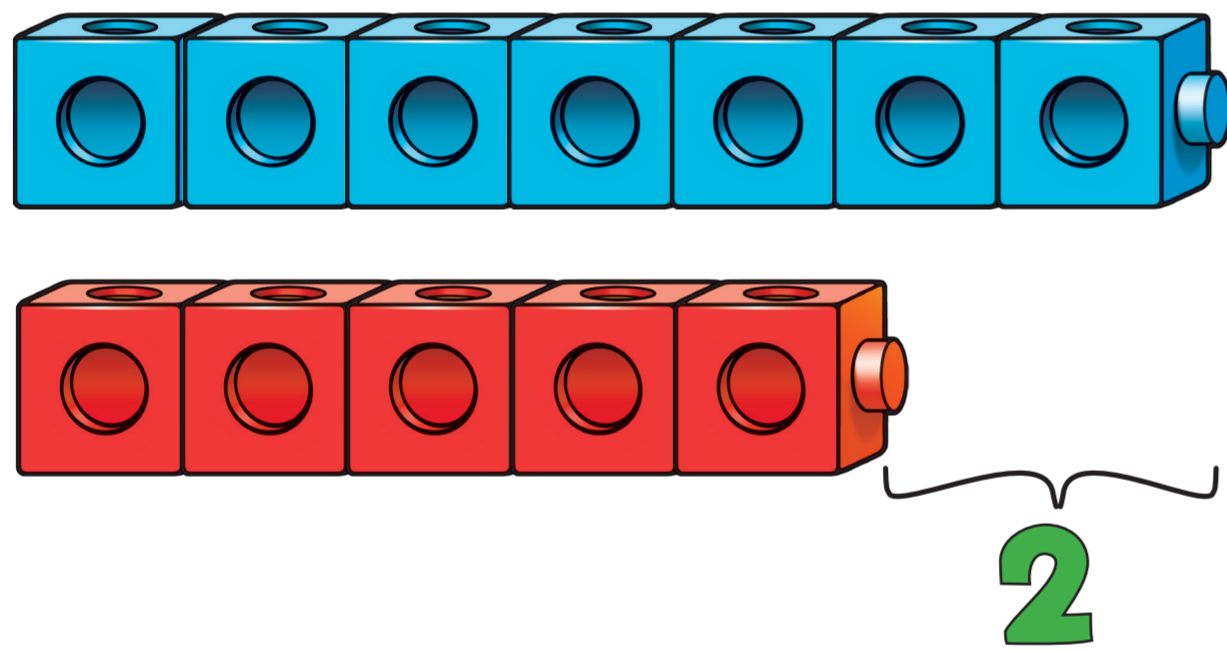
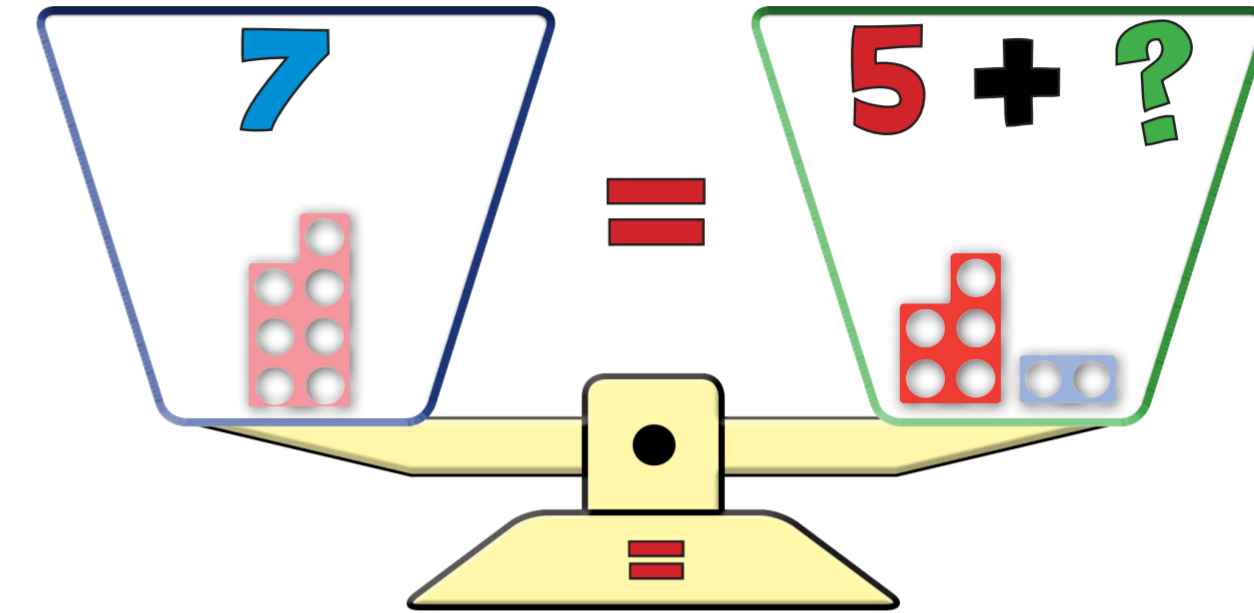
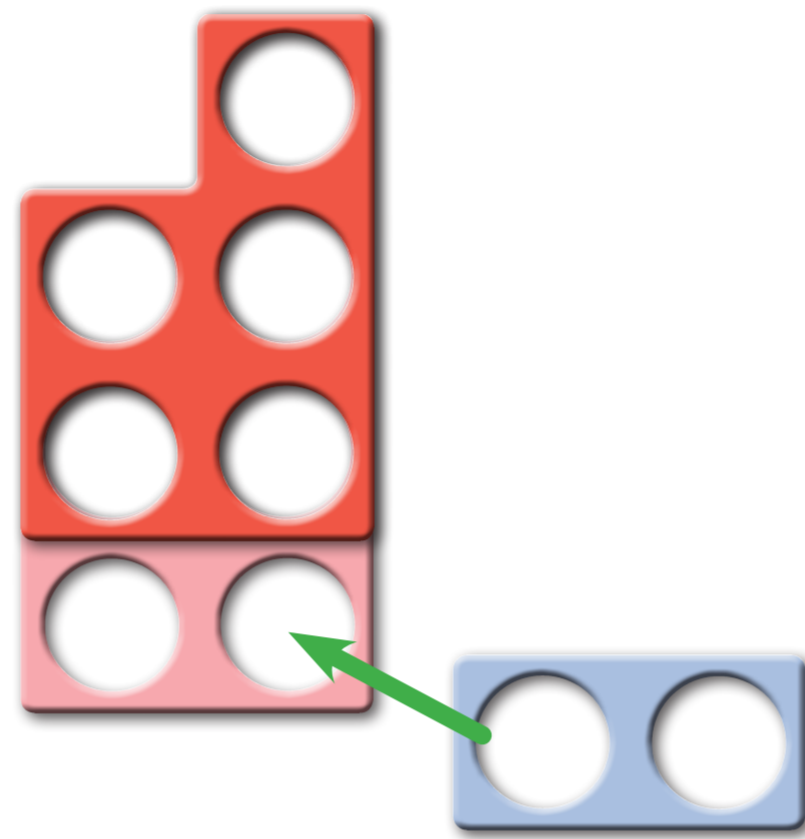
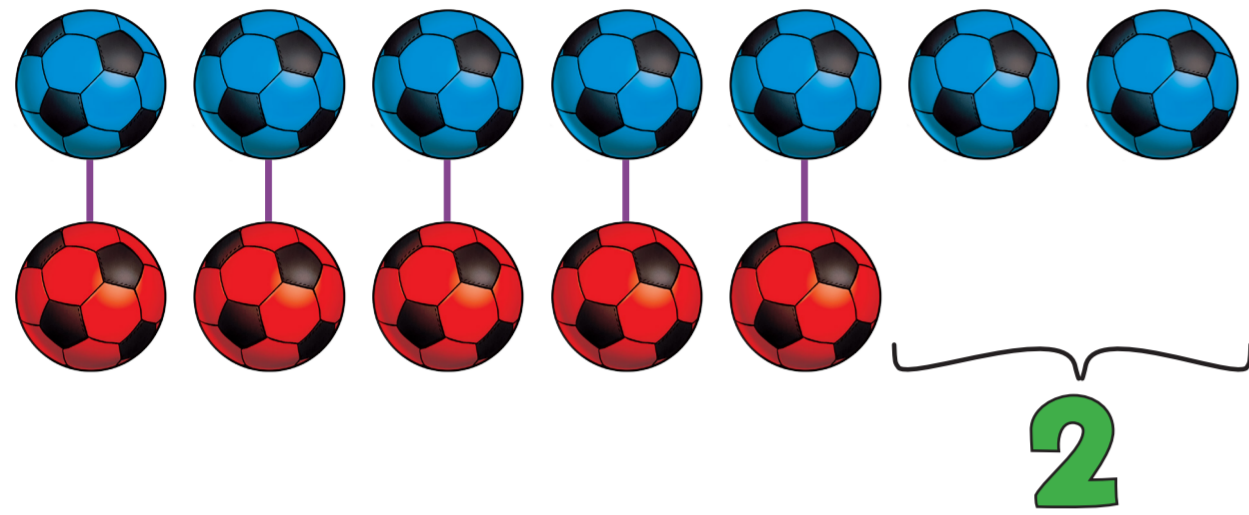
$$4 = 7 - 3$$



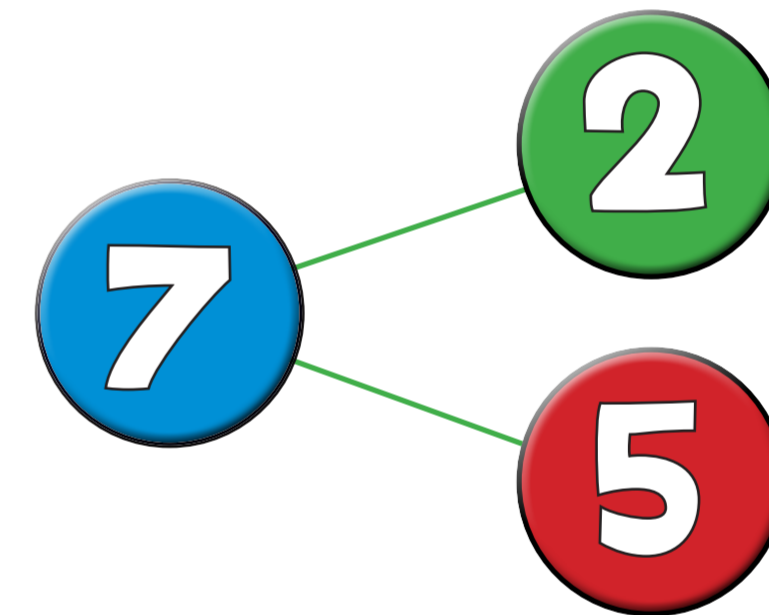
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30



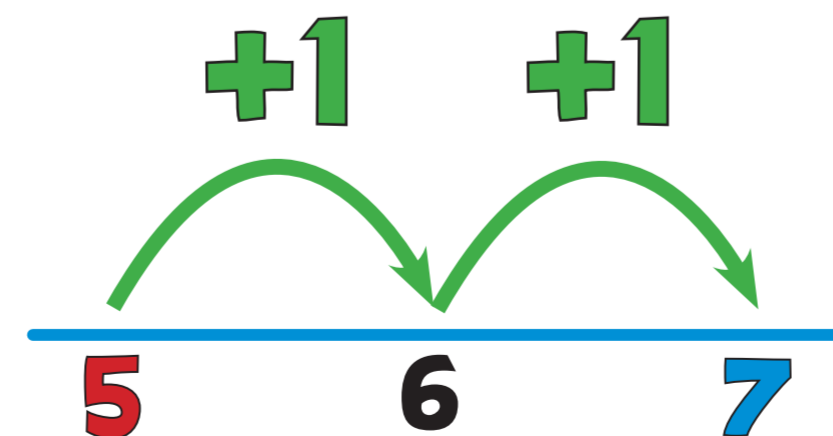
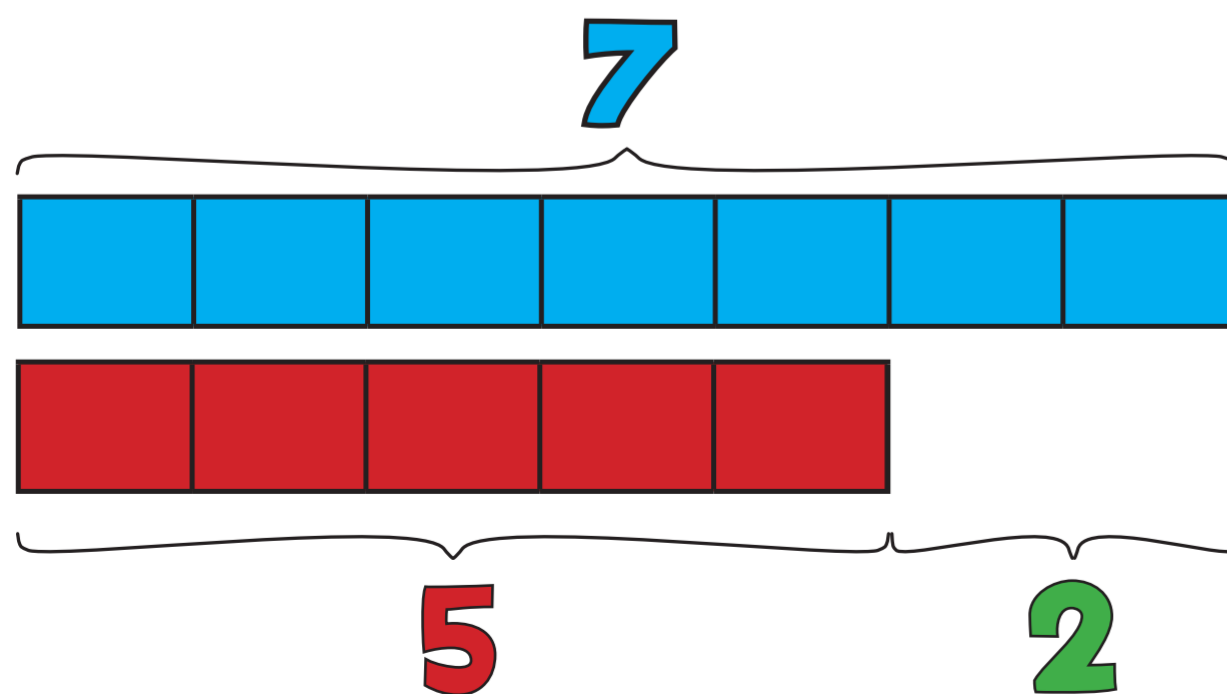
$$7 - 5 = 2$$



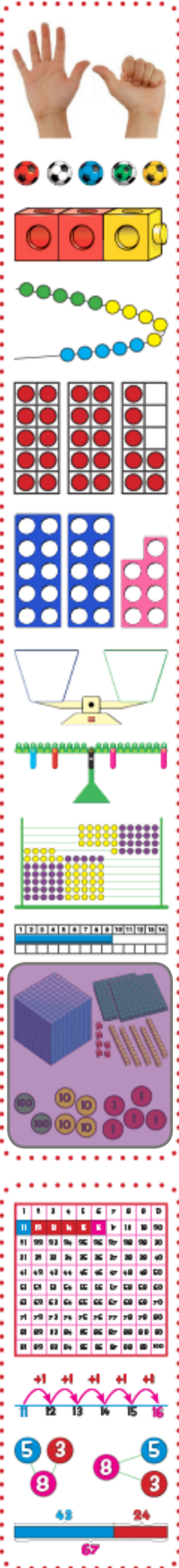
Count On
Images



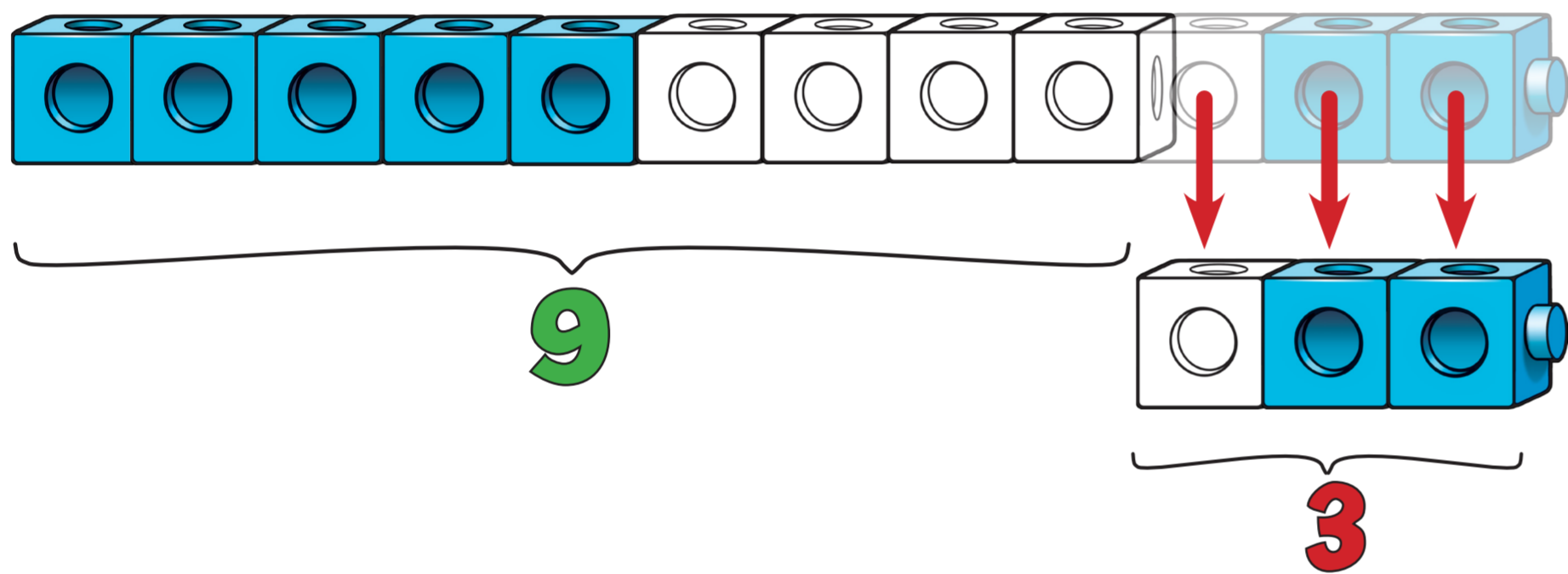
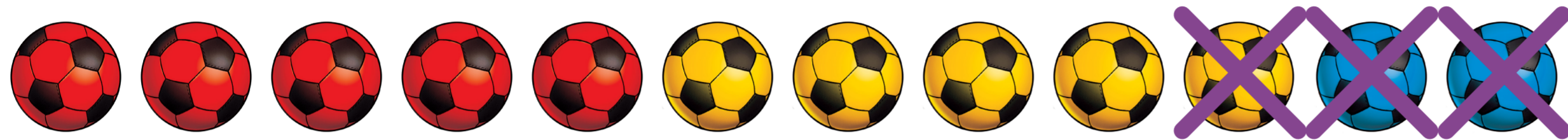
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11	12	13	14	15	16	17	18	19	20



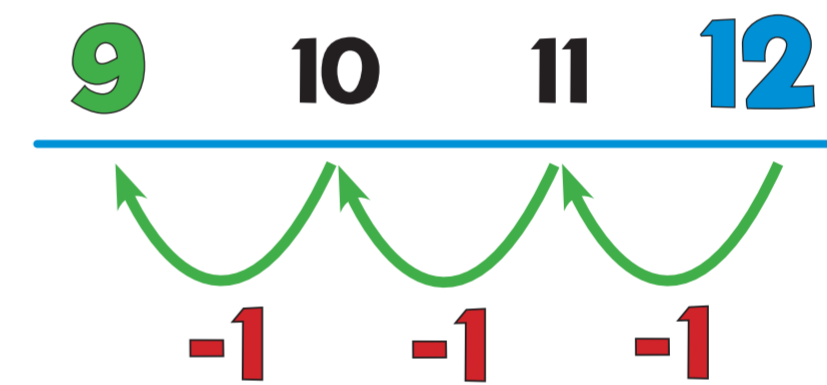
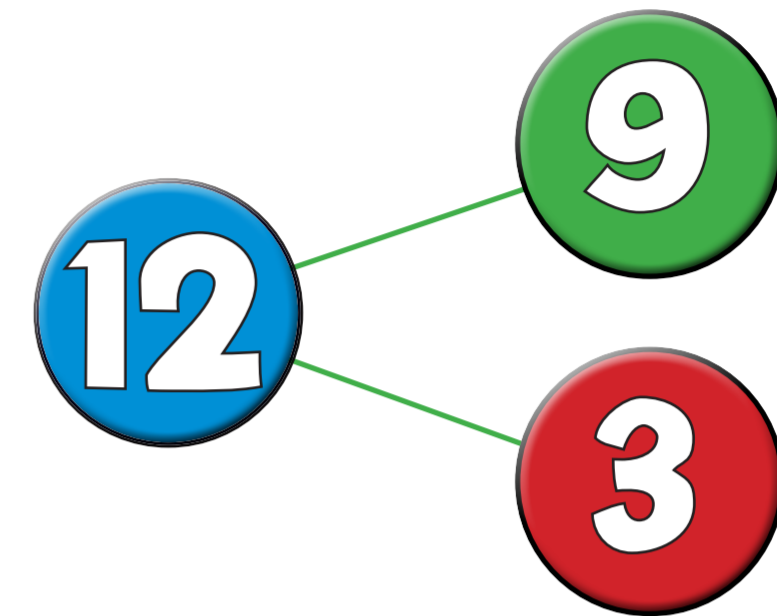
$$2 = 7 - 5$$



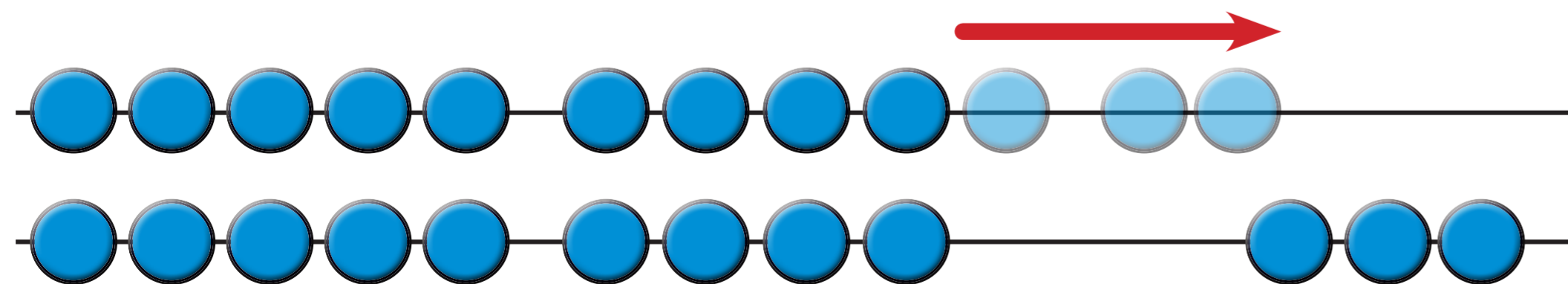
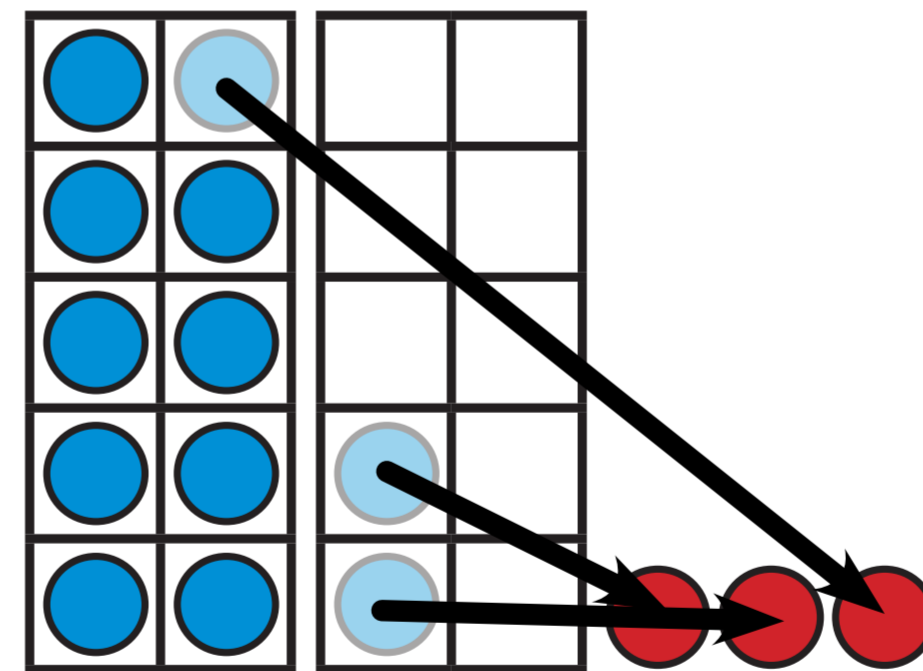
$$12 - 3 = 9$$



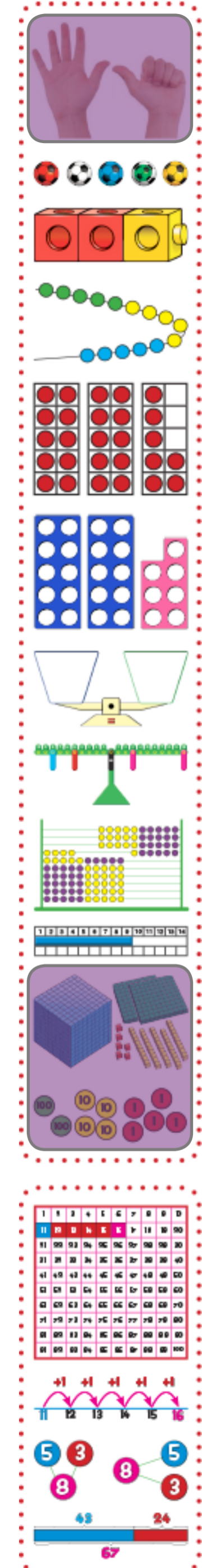
Count Back
Images



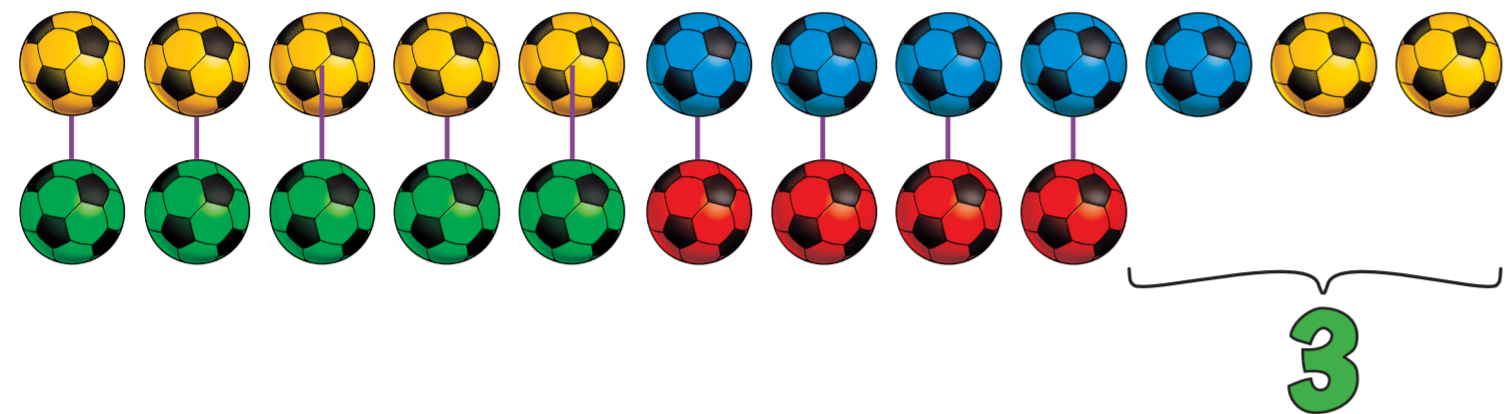
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30



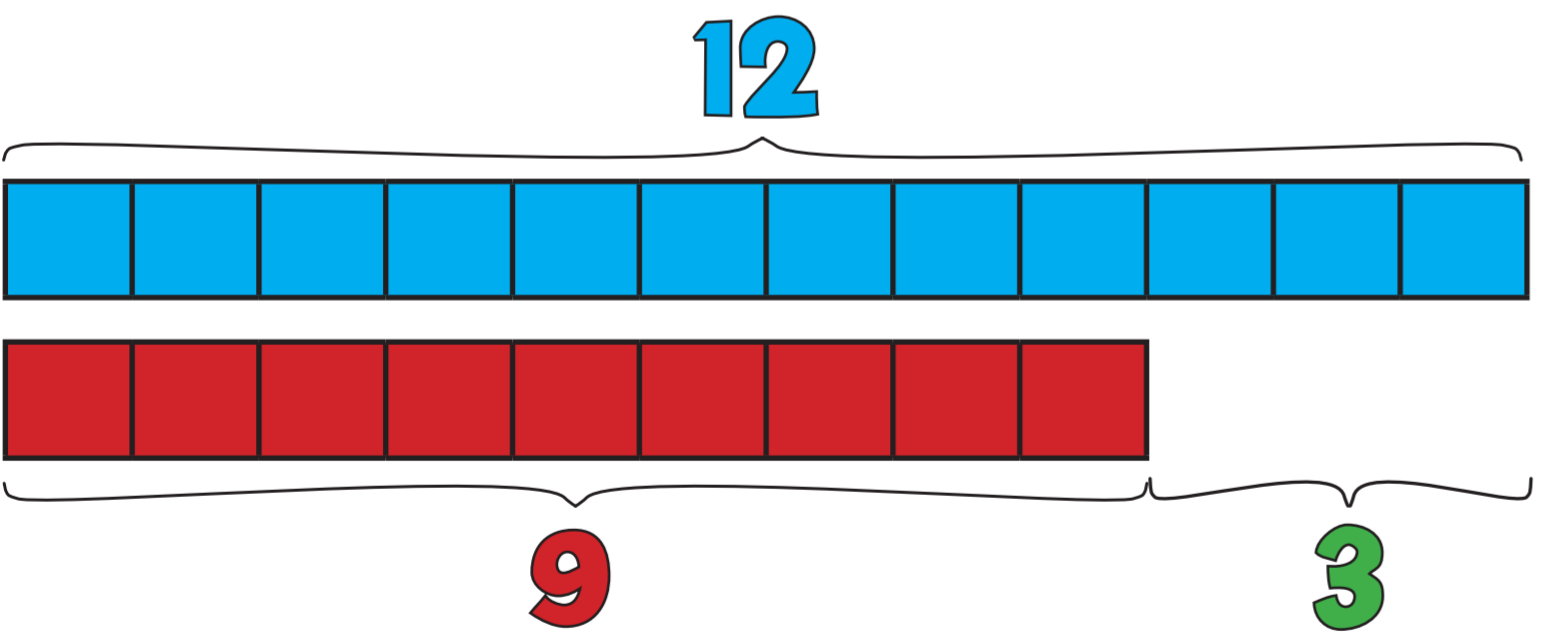
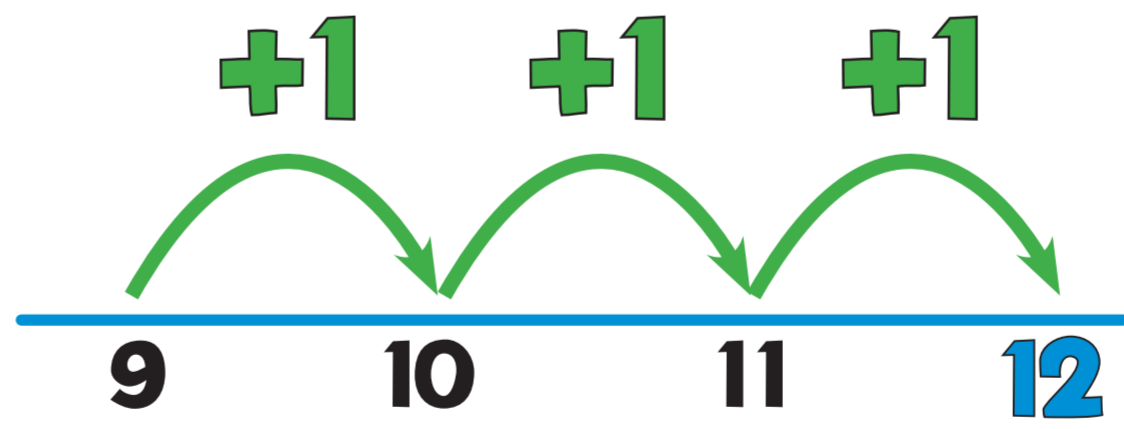
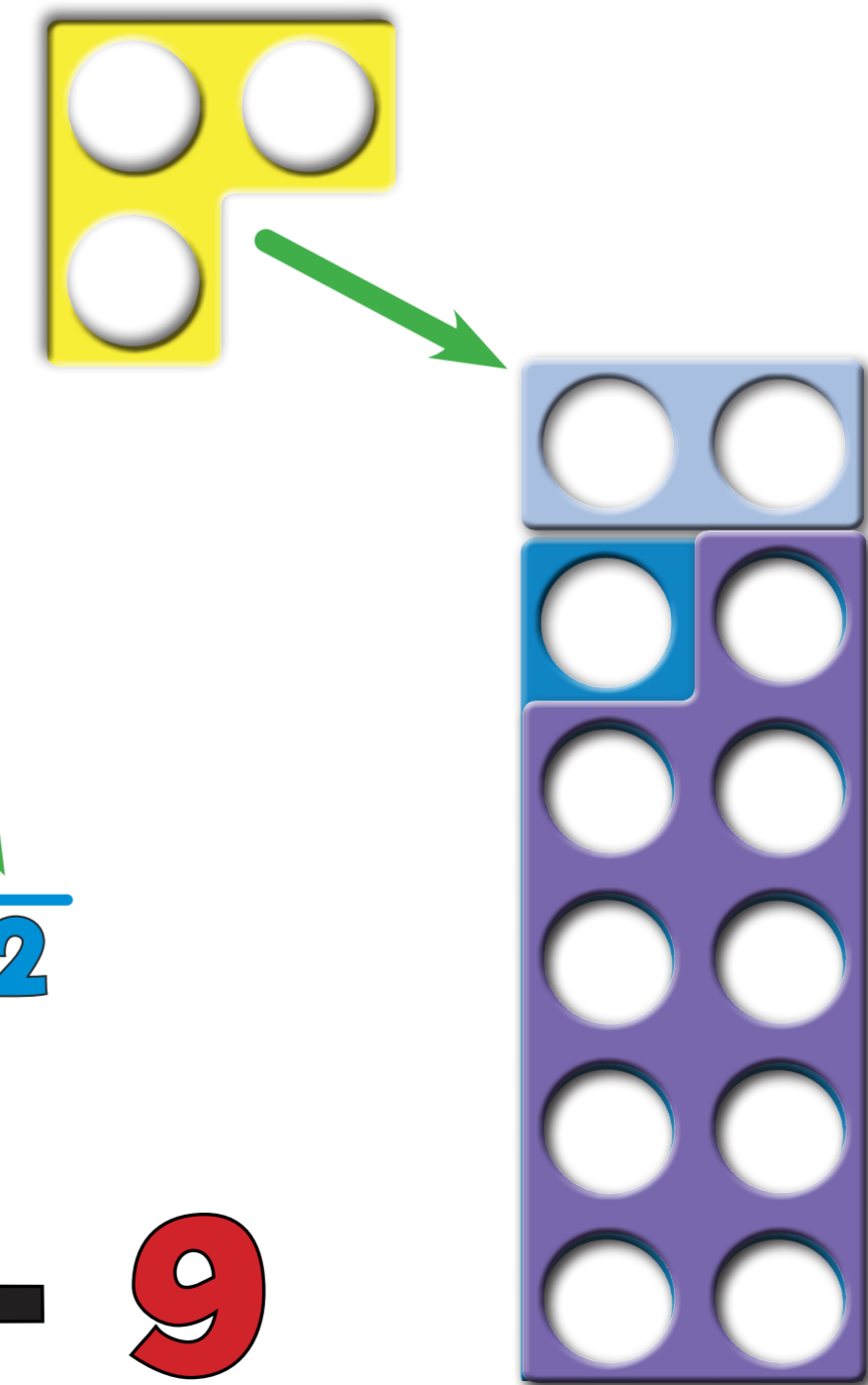
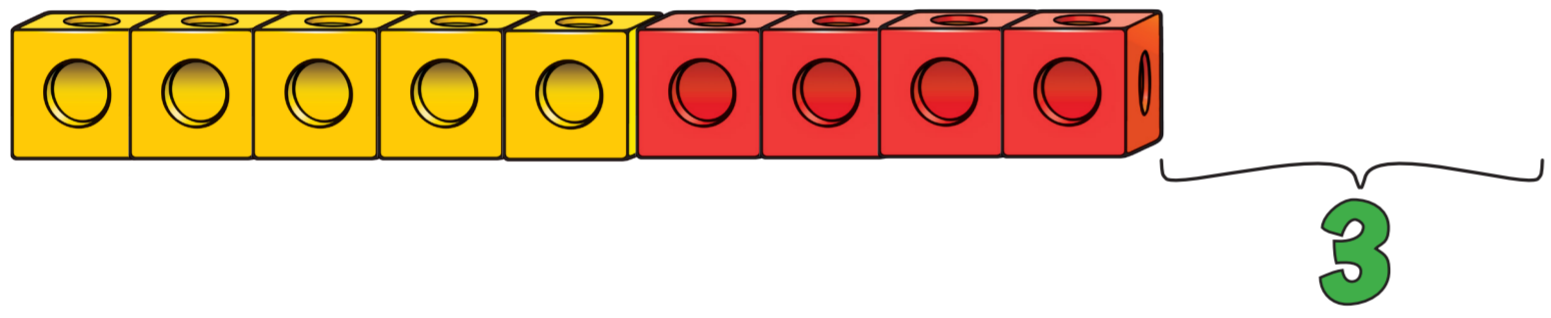
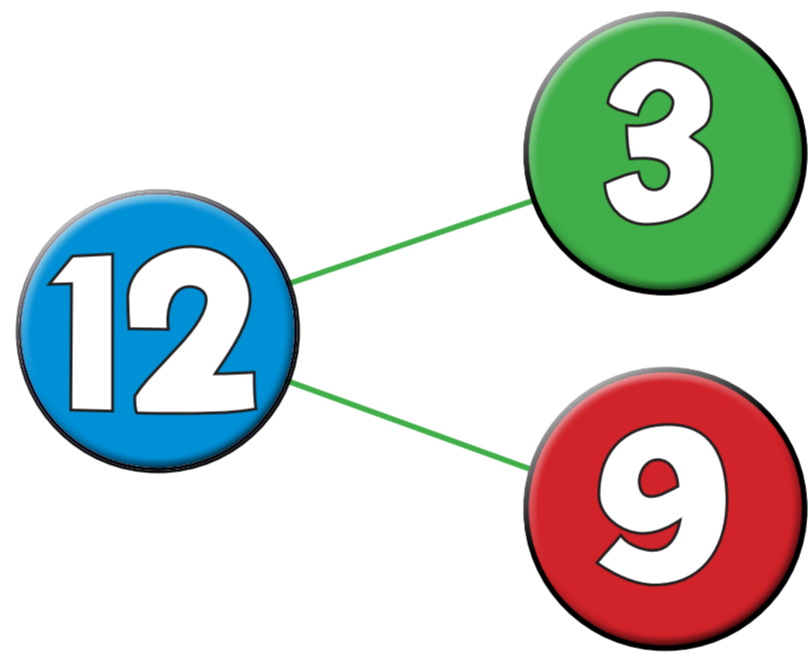
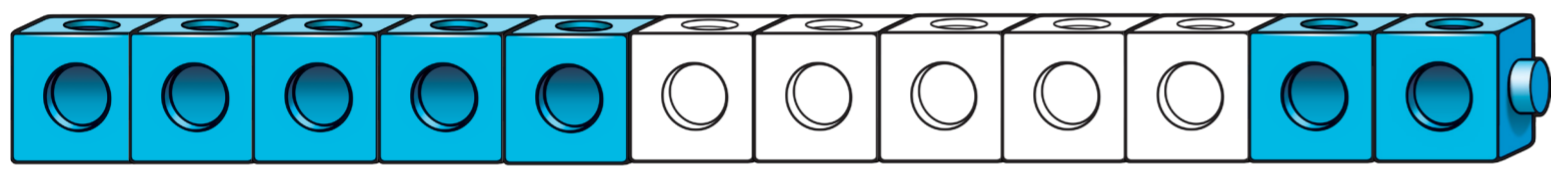
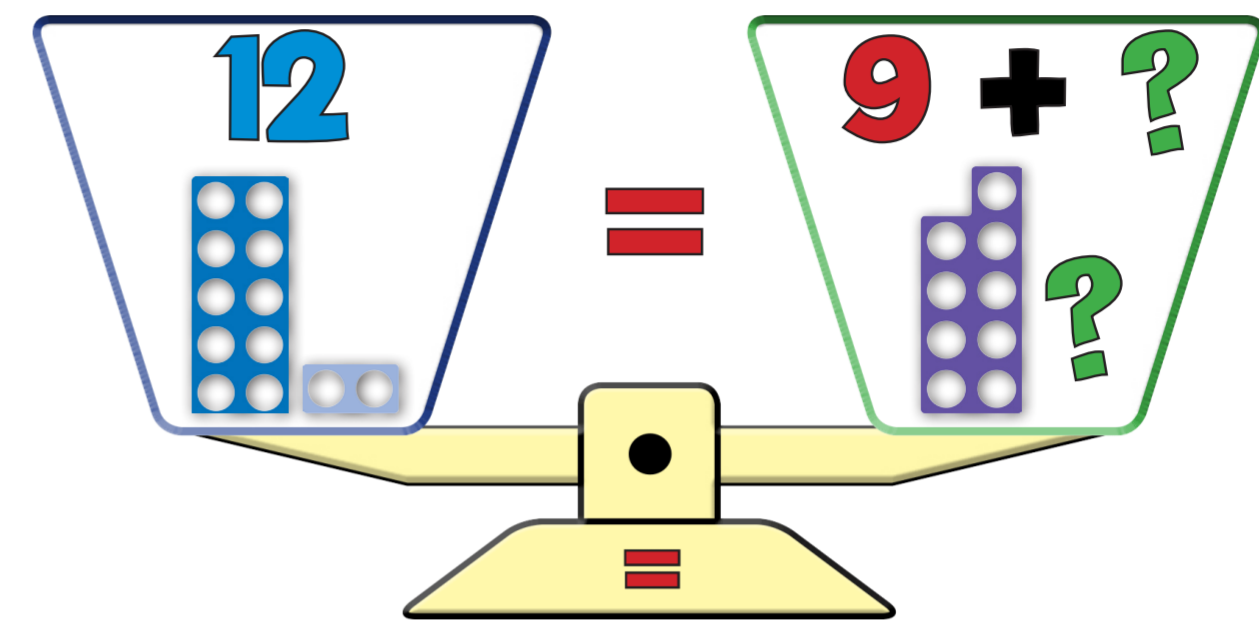
$$9 = 12 - 3$$



$$12 - 9 = 3$$

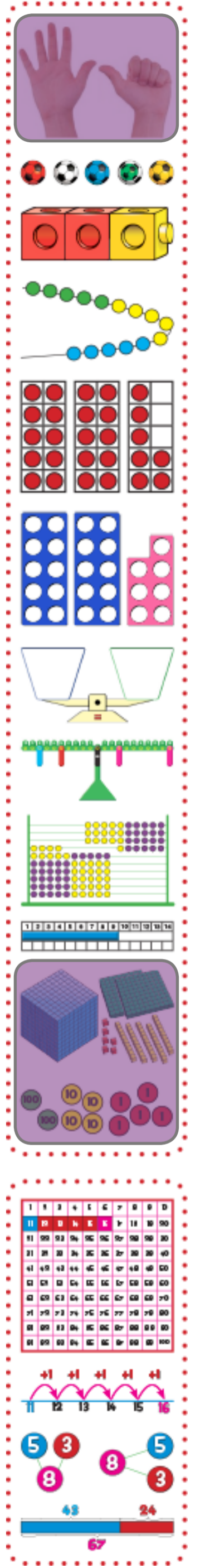


Count On Images



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20

$$3 = 12 - 9$$



$$87 - 23 = 64$$



1

10	1
8	7
-	23
<hr/>	

Tens: 8 yellow rods
Ones: 7 red cubes

2

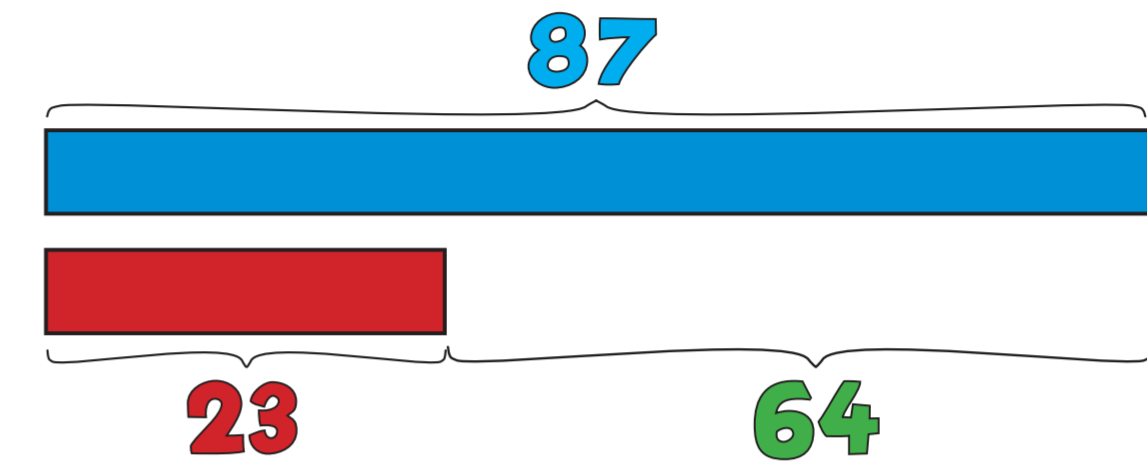
10	1
8	7
-	23
<hr/>	

Tens: 8 yellow rods
Ones: 7 red cubes, with 3 cubes being moved from the ones column to the tens column.

3

10	1
8	7
-	23
<hr/>	
	4

Tens: 5 yellow rods
Ones: 4 red cubes



4

10	1
8	7
-	23
<hr/>	
	4

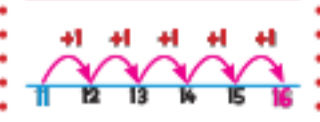
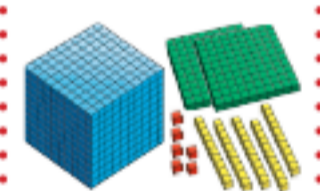
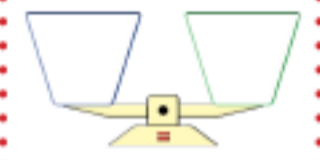
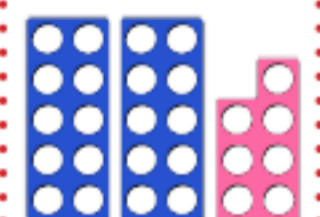
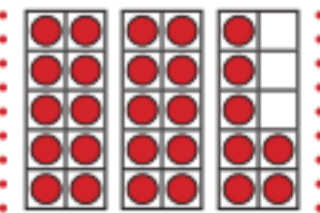
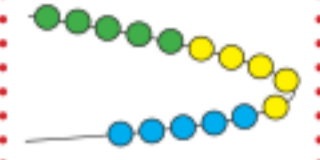
Tens: 5 yellow rods
Ones: 4 red cubes

5

10	1
8	7
-	23
<hr/>	
6	4

Tens: 6 yellow rods
Ones: 4 red cubes

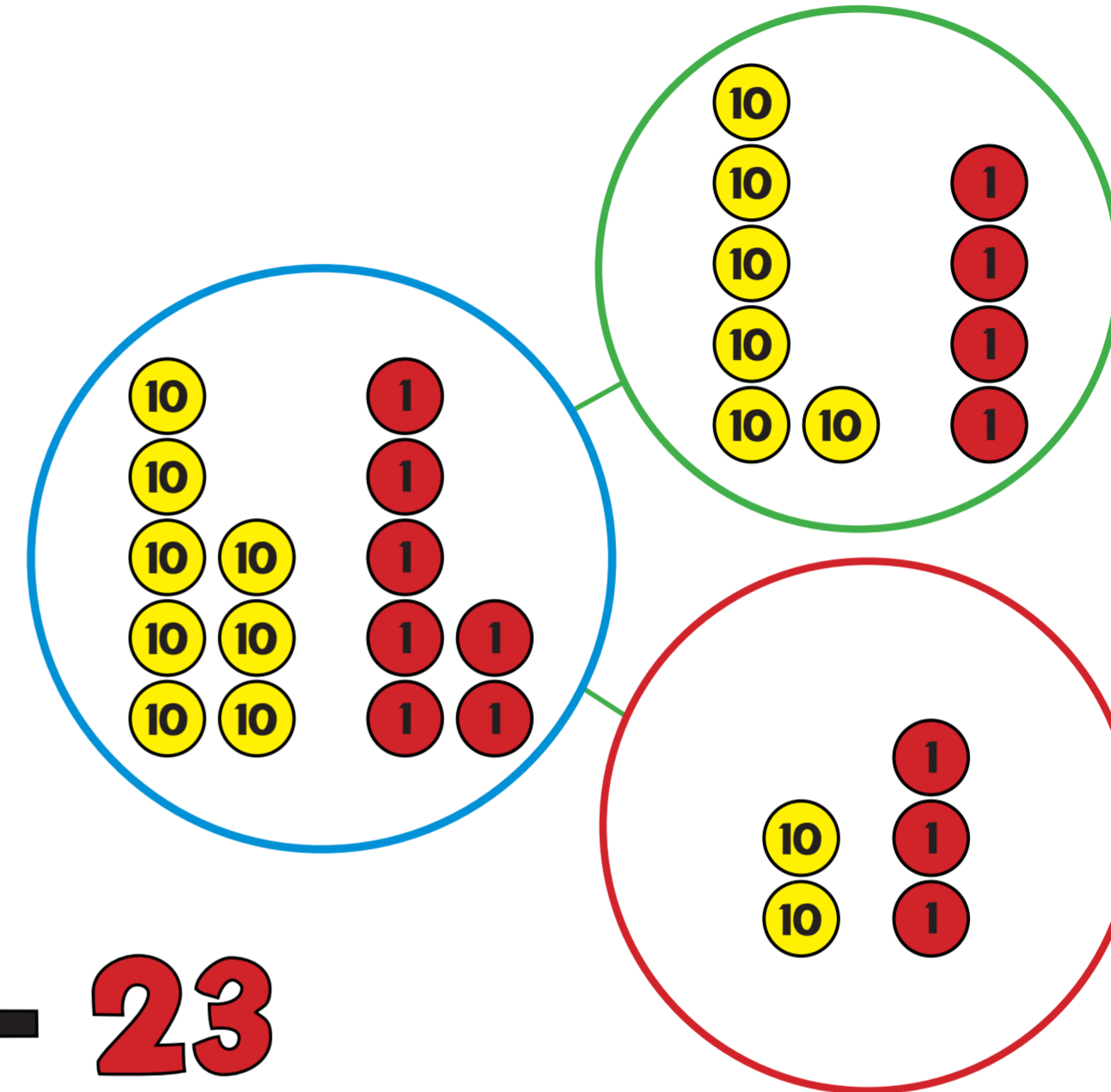
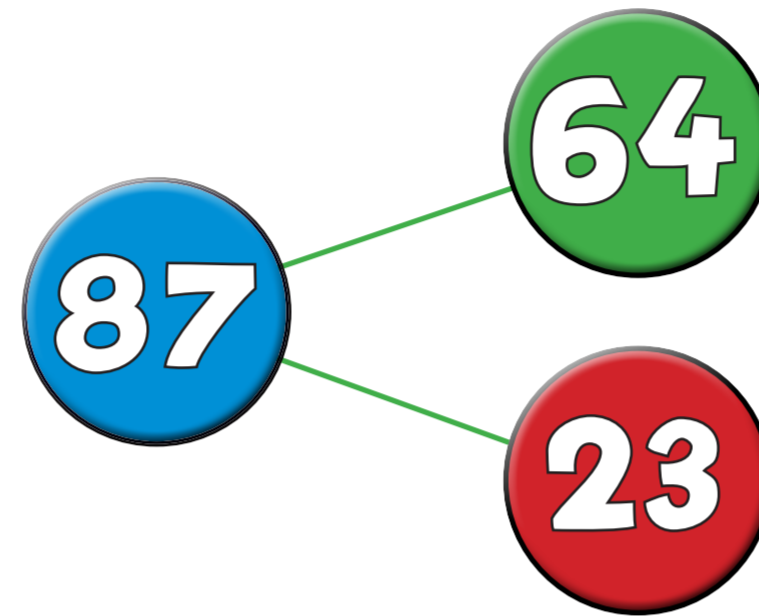
$$64 = 87 - 23$$



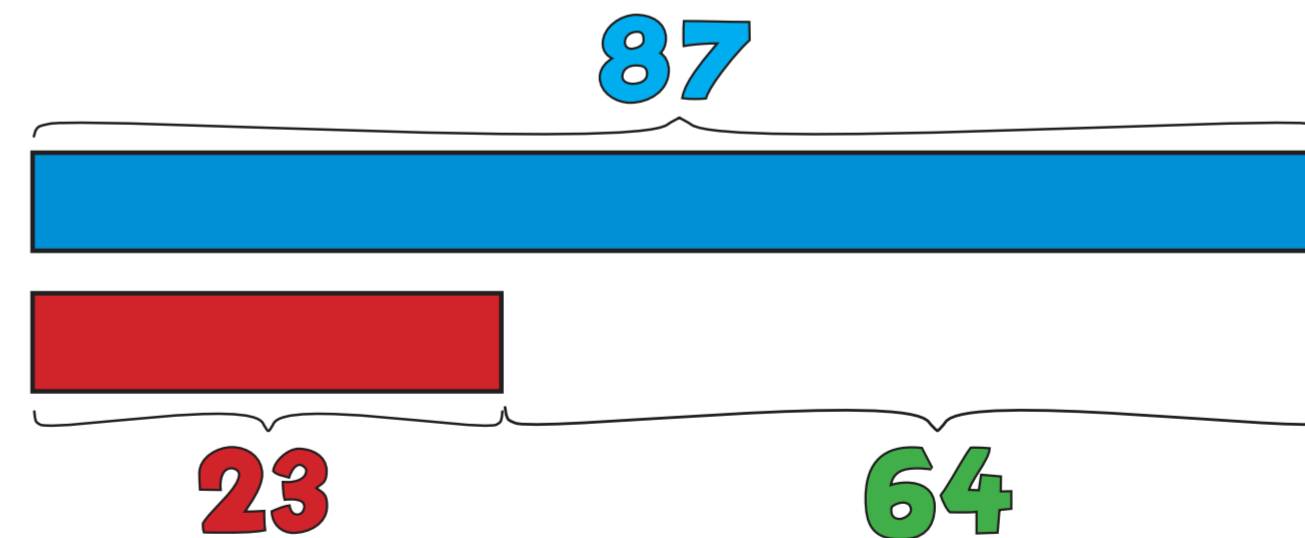
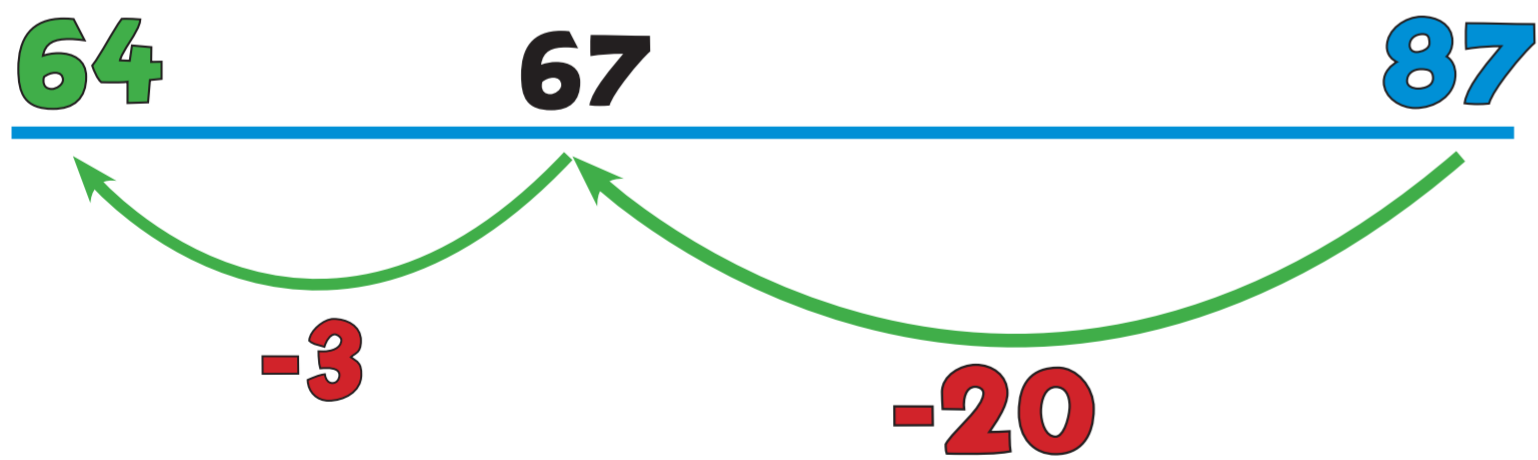
87 - 23 = 64



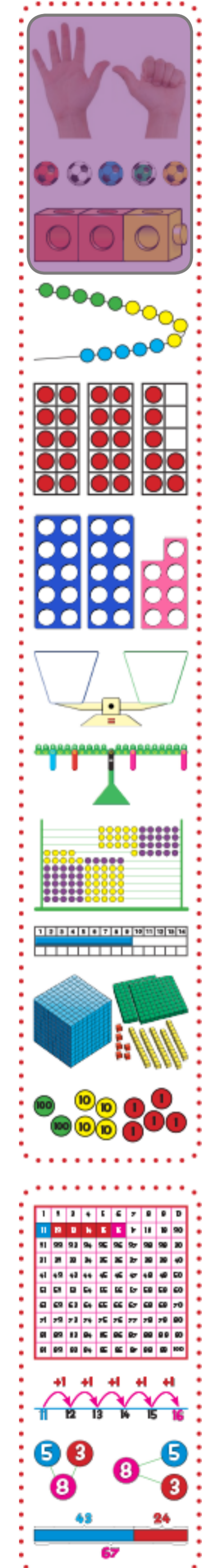
1	2	3	4	5	6	7	8	9	10
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21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



64 = 87 - 23



55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94



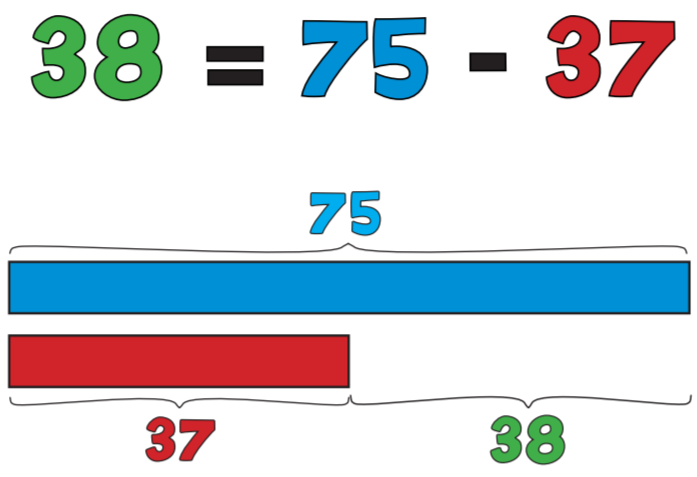
75 - 37 = 38



1

10	1
7	5
-	3
3	7
<hr/>	

Tens Ones



4

10	1
7 ⁶	5 ¹
-	3
3	7
<hr/>	
8	

Tens Ones

2

10	1
7 ⁶	5 ¹
-	3
3	7
<hr/>	

Tens Ones

5

10	1
7 ⁶	5 ¹
-	3
3	7
<hr/>	
8	

Tens Ones

3

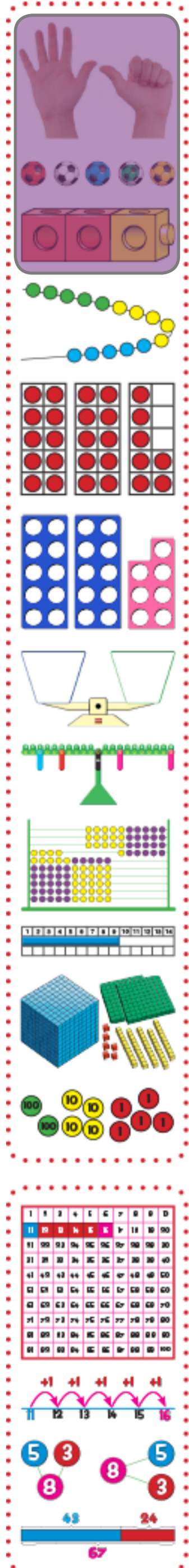
10	1
7 ⁶	5 ¹
-	3
3	7
<hr/>	

Tens Ones

6

10	1
7 ⁶	5 ¹
-	3
3	7
<hr/>	
38	

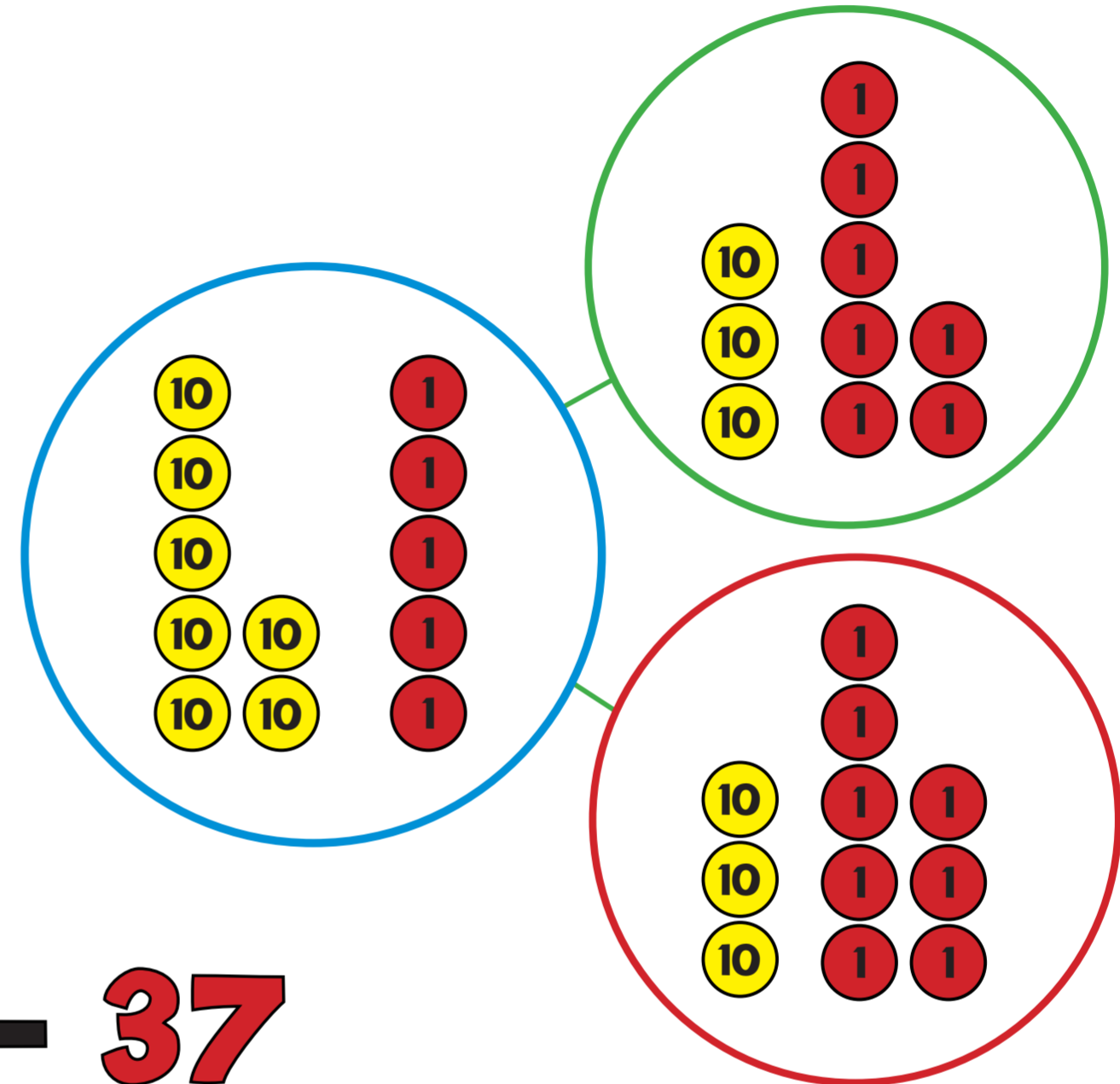
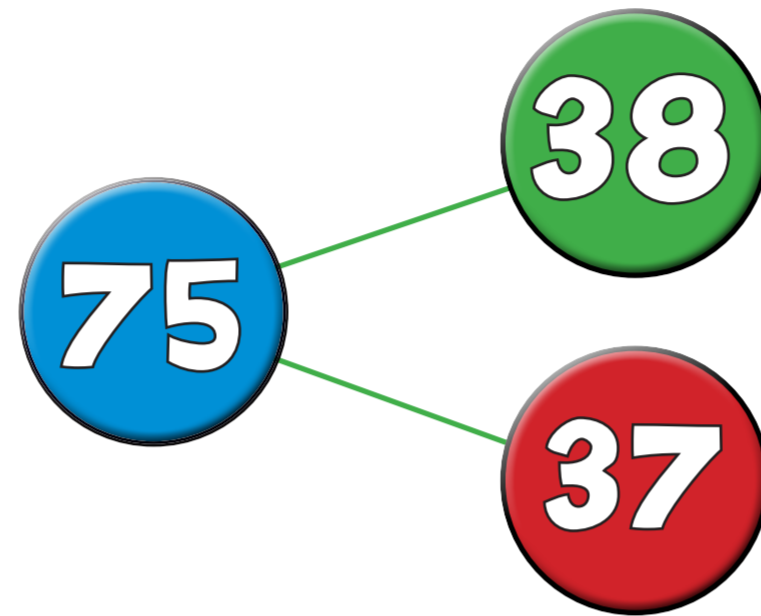
Tens Ones



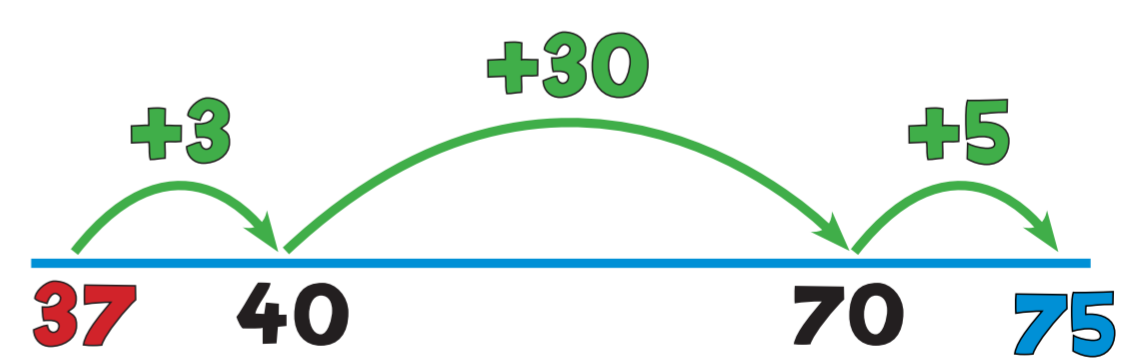
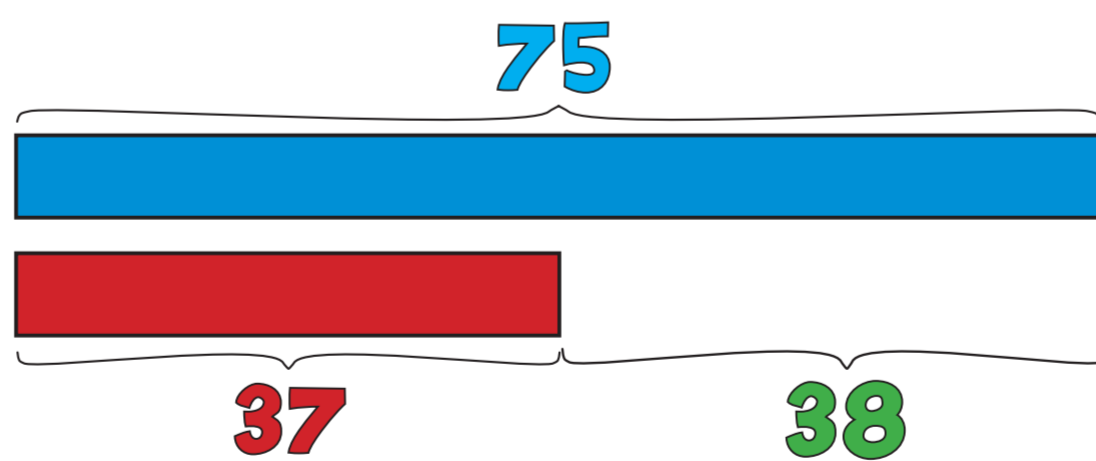
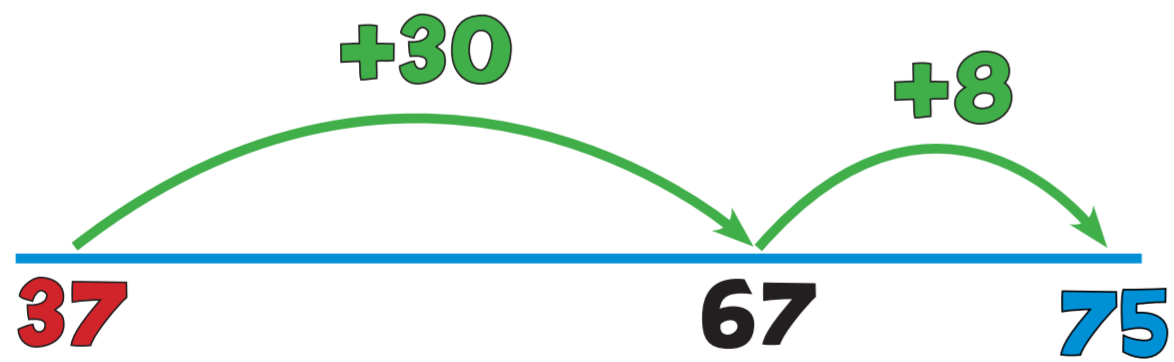
75 - 37 = 38



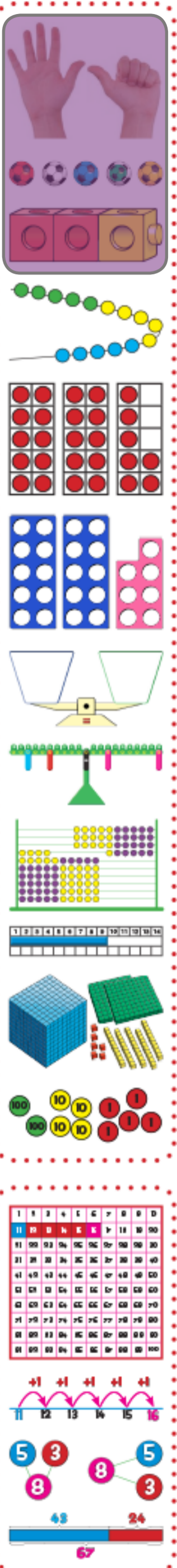
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



38 = 75 - 37



35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78



127 - 74 = 53

1

100	10	1
1	2	7
-	7	4
<hr/>		

Hundreds Tens Ones

2

100	10	1
1	2	7
-	7	4
<hr/>		

Hundreds Tens Ones

3

100	10	1
1	2	7
-	7	4
<hr/>		
		3

Hundreds Tens Ones

4

100	10	1
0	1	2
-	7	4
<hr/>		
		3

Hundreds Tens Ones

5

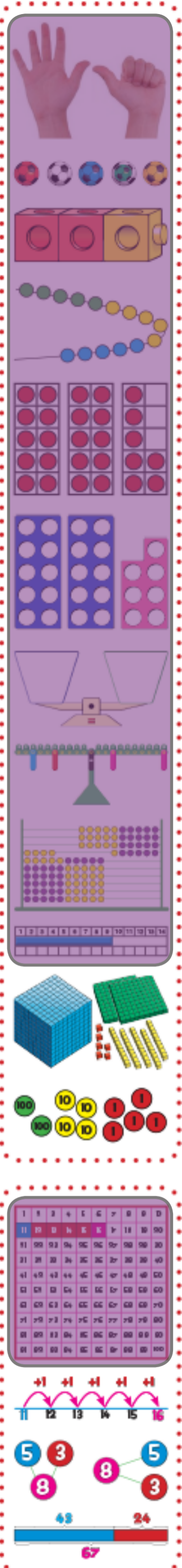
100	10	1
0	1	2
-	7	4
<hr/>		
		3

Hundreds Tens Ones

6

100	10	1
0	1	2
-	7	4
<hr/>		
5		3

Hundreds Tens Ones



127 - 74 = 53

1

100	10	1
1	2	7
-	7	4
<hr/>		
		3

Hundreds: 100 (green circle)
Tens: 10, 10 (yellow circles)
Ones: 1, 1, 1, 1, 1, 1 (red circles)

2

100	10	1
1	2	7
-	7	4
<hr/>		
		3

Hundreds: 100 (green circle)
Tens: 10, 10 (yellow circles)
Ones: 1, 1, 1, 1, 1, 1 (red circles, top two crossed out)

3

100	10	1
1	2	7
-	7	4
<hr/>		
		3

Hundreds: 100 (green circle)
Tens: 10, 10 (yellow circles)
Ones: 1, 1, 1 (red circles)

4

100	10	1
1	2	7
-	7	4
<hr/>		
		3

Hundreds: 100 (green circle)
Tens: 10, 10, 10, 10, 10, 10 (yellow circles)
Ones: 1, 1, 1 (red circles)

5

100	10	1
1	2	7
-	7	4
<hr/>		
		3

Hundreds: 100 (green circle)
Tens: 10, 10, 10, 10, 10, 10 (yellow circles, top two crossed out)
Ones: 1, 1, 1 (red circles)

6

100	10	1
1	2	7
-	7	4
<hr/>		
	5	3

Hundreds: 100 (green circle)
Tens: 10, 10, 10, 10, 10 (yellow circles)
Ones: 1, 1, 1 (red circles)

